Math 2214 **Common Final Exam** **Spring 2017**

Form A

Instructions: Fill in A, B or C in the Test Version section. Then enter your NAME, ID Number, CRN (under Class ID) and write A, B, or C (under Test ID) on the op-scan sheet. Darken the appropriate circles below your ID number and Class ID (CRN). **Use a number 2 pencil**. Machine grading may ignore faintly marked circles.

Mark your answers to the test questions in rows 1–15 of the op-scan sheet. Your score on this test will be the number of correct answers. You have one hour to complete this portion of the exam. Turn in the op-scan sheet with your answers, this exam and all scrap paper at the end of this part of the final exam.

Exam Policies: You may not use a book, notes, formula sheet, or a calculator or computer. Giving or receiving unauthorized aid is an Honor Code Violation.

Signature: ______________________________

Name (printed): ______________________________

Student ID #: ______________________________

1. Determine the largest interval on which the initial value problem

$$(t-3)y'' + \frac{y'}{t-5} - \frac{3y}{t} = \frac{7}{\cos(t)}, \qquad y(2) = 8,\ y'(2) = 3$$

is guaranteed to have a unique solution. **DO NOT attempt to find the soluton**

(A) $\left(\frac{\pi}{2}, 3\right)$ (B) $\left(\frac{\pi}{2}, 5\right)$ (C) $(0, 3)$ (D) $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

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2. What is the integrating factor for the differential equation

$$\frac{y'}{t} - \frac{2y}{t^2} = t\cos(t) \qquad t > 0$$

(A) $\mu(t) = -2\ln(t)$ (B) $\mu(t) = \frac{1}{t^2}$ (C) $\mu(t) = e^{-2/t}$ (D) $\mu(t) = \frac{2}{t}$

3. Rewrite the third-order differential equation $y''' + t^2 y'' = (\sin t)y$ as a linear system of first-order differential equations.

(A) $\mathbf{y}'(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\sin t & 0 & t^2 \end{bmatrix} \mathbf{y}(t)$

(B) $\mathbf{y}'(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ t^2 & 0 & -\sin t \end{bmatrix} \mathbf{y}(t)$

(C) $\mathbf{y}'(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \sin t & 0 & -t^2 \end{bmatrix} \mathbf{y}(t)$

(D) $\mathbf{y}'(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -t^2 & 0 & \sin t \end{bmatrix} \mathbf{y}(t)$

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4. The general solution of

$$y' - t^2 \frac{e^{-y^2}}{y} = 0$$

is

(A) $\frac{1}{2} e^{y^2} = \frac{t^3}{3} + C$

(B) $\frac{e^{y^2}}{2y} = \frac{t^3}{3} \ln|y| + C$

(C) $y^2 e^{y^2} = C - \frac{t^3}{3}$

(D) $y^2 = \ln\left|\frac{t^3}{3} + C\right|$

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5. What are the possible values for the sum $x_0 + y_0$, if a point (x_0, y_0) is an equilibrium solution of the following system of equations?

$$\begin{aligned} x' &= y^2 - 2xy \\ y' &= 6y + x^2 \end{aligned}$$

(A) 0 only

(B) 0 or -12

(C) 0 or -36

(D) 0, -12, or -36

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6. Which of the following statements are true

I. The differential equation $y' + e^{ty} = e^y \sin(t)$ is separable.

II. The differential equation $4ty + y'y = 1$ is nonlinear.

III. The differential equation $y' = \tan\left(y^2 - 5y - 6\right)$ has an equilibrium solution $y = 6$.

(A) Only (II) and (III)

(B) Only (I) and (III)

(C) Only (I) and (II)

(D) (I), (II), and (III)

7. Suppose the solution to a spring system is

$$y(t) = 4e^{-2t} + 5te^{-2t}.$$

Then the characteristic equation must be

(A) $(\lambda - 4)(\lambda - 5) = 0$

(B) $(\lambda + 2)(\lambda - 2) = 0$

(C) $(\lambda + 2)(\lambda - 2) = 0$

(D) $(\lambda + 2)(\lambda + 2) = 0$

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8. The trial form for the particular solution of $y'' + 2y' = 9e^t + 6t$ is

(A) $Ae^t + Bt + C$

(B) $Ae^t + Bt^2 + Ct$

(C) $Ate^t + Bt + C$

(D) $Ate^t + Bt^2 + Ct$

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9. Suppose that a 3×3 matrix A in the system $\mathbf{y}' = A\mathbf{y}$ has the following eigenpairs

$$\lambda_1 = 2, \mathbf{x}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \qquad \lambda_2 = 2, \mathbf{x}_2 = \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}, \qquad \lambda_3 = -1, \mathbf{x}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

If the fundamental matrix $\Psi(t)$ is constructed using these eigenvectors, its Wronskian is (here K is a nonzero constant):

(A) $W(t) = Ke^{-3t}$

(B) $W(t) = Ke^{-4t}$

(C) $W(t) = Ke^{4t}$

(D) $W(t) = Ke^{3t}$

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10. Suppose that a 3×3 linear system $\mathbf{y}' = A\mathbf{y}$ has an eigenvalue $\lambda = 2 + 5i$ with eigenvector $\begin{bmatrix} 1+i \\ 1-i \\ 2 \end{bmatrix}$.

Select a pair of real valued solutions that we can obtain with this information.

$$\textbf{A:}\ e^{2t}\begin{bmatrix} \cos(5t) + \sin(5t) \\ \sin(5t) - \cos(5t) \\ 2\cos(5t) \end{bmatrix} \quad \textbf{B:}\ e^{2t}\begin{bmatrix} \cos(5t) - \sin(5t) \\ \cos(5t) + \sin(5t) \\ 2\cos(5t) \end{bmatrix}$$

$$\textbf{C:}\ e^{2t}\begin{bmatrix} \cos(5t) + \sin(5t) \\ \sin(5t) - \cos(5t) \\ 2\sin(5t) \end{bmatrix} \quad \textbf{D:}\ e^{2t}\begin{bmatrix} \cos(5t) - \sin(5t) \\ \cos(5t) + \sin(5t) \\ 2\sin(5t) \end{bmatrix}$$

(A) **B** and **C**

(B) **B** and **D**

(C) **A** and **D**

(D) **A** and **C**

11. A 2×2 matrix A has a repeated eigenvalue $\lambda = 2$. If the matrix has an eigenvector $\mathbf{x} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and a corresponding generalized eigenvector $\mathbf{v} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, then the general solution to the first order system $\mathbf{y}' = A\mathbf{y}$ has the form

(A) $\mathbf{y} = c_1 \begin{bmatrix} 2e^{2t} \\ e^{2t} \end{bmatrix} + c_2 \begin{bmatrix} (2+t)e^{2t} \\ (1-t)e^{2t} \end{bmatrix}$

(B) $\mathbf{y} = c_1 \begin{bmatrix} 2e^{2t} \\ e^{2t} \end{bmatrix} + c_2 \begin{bmatrix} (2t+1)e^{2t} \\ (t-1)e^{2t} \end{bmatrix}$

(C) $\mathbf{y} = c_1 \begin{bmatrix} 2e^{2t} \\ e^{2t} \end{bmatrix} + c_2 \begin{bmatrix} te^{2t} \\ -te^{2t} \end{bmatrix}$

(D) $\mathbf{y} = c_1 \begin{bmatrix} 2e^{2t} \\ e^{2t} \end{bmatrix} + c_2 \begin{bmatrix} e^{2t} \\ -e^{2t} \end{bmatrix}$

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12. Consider the linear system $\mathbf{y}' = \begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix} \mathbf{y}$. Classify the stability of the equilibrium point $\mathbf{y}_e = \mathbf{0}$.

(A) The origin is an asymptotically stable equilibrium point.

(B) The origin is a stable, but not asymptotically stable, equilibrium point.

(C) The origin is an unstable equilibrium point.

(D) The origin is not an equilibrium point at all.

13. The general solution to

$$y'' + y = \sec(t)$$

is:

(A) $c_1 \cos(t) + c_2 \sin(t) - \sin(t) \int \tan(t)\, dt + \cos(t) \int 1\, dt$

(B) $c_1 \cos(t) + c_2 \sin(t) - \int \tan(t)\, dt + \int 1\, dt$

(C) $c_1 \cos(t) + c_2 \sin(t) - \cos(t) \int \tan(t)\, dt + \sin(t) \int 1\, dt$

(D) $c_1 \cos(t) + c_2 \sin(t) - 1 + \dfrac{\sin(t)}{\cos(t)}$

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14. Find the linearization $\mathbf{y}' = A\mathbf{y}$ of the system

$$\begin{aligned} x_1{}' &= x_1 + x_2 - 2 \\ x_2{}' &= x_1{}^2 - x_2{}^2 + 4 \end{aligned}$$

at its equilibrium point $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$.

(A) $\mathbf{y}' = \begin{bmatrix} 1 & 1 \\ 0 & 4 \end{bmatrix} \mathbf{y}$

(B) $\mathbf{y}' = \begin{bmatrix} 1 & 1 \\ 0 & -4 \end{bmatrix} \mathbf{y}$

(C) $\mathbf{y}' = \begin{bmatrix} 1 & 1 \\ 0 & -2 \end{bmatrix} \mathbf{y}$

(D) $\mathbf{y}' = \begin{bmatrix} 1 & 0 \\ 1 & 4 \end{bmatrix} \mathbf{y}$