

1. Which of the following is a particular solution of the equation  $y'' - y = e^t$ ?

(a)  $te^t/2 + e^{-t}$ .

(b)  $e^t/2$ .

(c)  $te^t + 5e^t$ .

(d)  $2e^t + 3e^{-t}$ .

2. A nonlinear system is given by

$$x_1' = x_1^2 - x_2^3 x_1.$$

$$x_2' = x_2 - x_1.$$

How many equilibrium points does this system have?

(a) 5.

(b) 2.

(c) 6.

(d) 3.

3. A mass of 5 kg stretches a spring by 9.8 cm in equilibrium. You subject the same mass-spring system to oscillatory forcing. You should expect particularly large oscillations when the period of the oscillation is close to

(a)  $\pi/5$  seconds.

(b)  $7\pi/5$  seconds.

(c) 9.8 seconds.

(d) 0.1 seconds.

4. The general solution of the system  $y' = Ay$ , where

$$A = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix},$$

is

(a)  $c_1 e^{3t} \cos t \begin{pmatrix} 2 \\ 3 \end{pmatrix} + c_2 e^{3t} \sin t \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ .

(b)  $c_1 e^{3t} \cos t \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{3t} \sin t \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$

(c)  $c_1 e^{3t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^t \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$

(d)  $c_1 e^{4t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$

5. You solve the initial value problem  $y' = 3 + y^2$ ,  $y(1) = 1$ , using the Euler method with  $h = 0.05$ . Then the approximation you find for  $y(1.1)$  is

(a) 1.422.

(b) 1.2.

(c) 1.4.

(d) 1.386.

6. A water tank initially contains 40 gallons of water in which 200 grams of salt are dissolved. Water enters and leaves the tank at a rate of 3 gallons per minute. The incoming solution contains 6 grams of salt per gallon, and the outgoing solution is well-mixed. The amount of salt in the tank, measured in grams, with time measured in minutes, is given by

(a)  $Q(t) = 200 \exp(-3t/40) + 18t.$

(b)  $Q(t) = 240 - 40 \exp(-3t/40).$

(c)  $Q(t) = 200 + 18t.$

(d)  $200 + 18 \exp(-3t/40).$

7. The general solution of the equation  $y'''' - y' = 0$  is

(a)  $y = A + Be^t + Cte^t + Dt^2e^t.$

(b)  $y = Ae^t + Be^{-t} + C + Dt.$

(c)  $y = Ae^t + Be^{-t} + C \cos t + D \sin t.$

(d)  $y = A + Be^t + Ce^{-t/2} \cos(\sqrt{3}t/2) + De^{-t/2} \sin(\sqrt{3}t/2).$

8. The geometric multiplicity of the eigenvalue  $-1$  for the matrix

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 2 & -1 & 0 & 0 \\ 3 & 0 & -1 & 0 \\ 4 & 2 & 0 & -1 \end{pmatrix}$$

is

- (a) 2.
- (b) 1.
- (c) 3.
- (d) 4.

9. For the system

$$\begin{aligned} x' &= -3x + 5y, \\ y' &= -5x + 4y, \end{aligned}$$

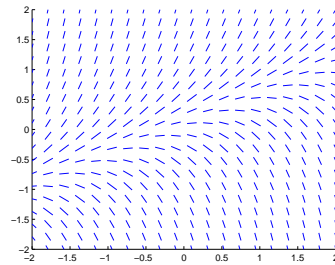
the origin is a(n)

- (a) stable focus.
- (b) stable node.
- (c) unstable focus.
- (d) saddle.

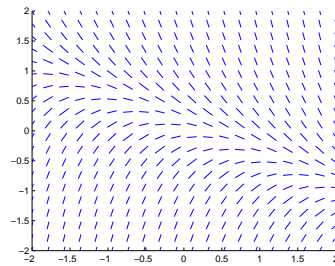
10. Which of the following is a linear equation?

- (a)  $y'' + \sin y = 0$ .
- (b)  $y' + y = \cos y$ .
- (c)  $y''/y' = \ln(\cos t)$ .
- (d)  $y''' - 4y^2 = \cos t$ .

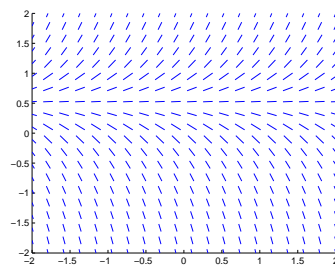
11. Which of the following is a direction field for the equation  $y' = 2y - t$ ?



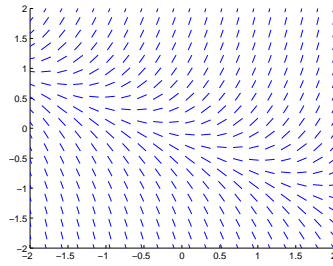
(a)



(b)



(c)



(d)

12. If  $x' = tx^2$ , and  $x(0) = 1/2$ , then  $x(1)$  is

- (a)  $\sqrt{\exp(1 - \ln 2)}$ .
- (b) 1.
- (c)  $2/3$ .
- (d)  $e/2$ .