Math 2214, Fall 2016, Form A

1. A nonlinear system is given by

$$x_1' = x_2^2 - x_1 x_2.$$

$$x_2' = x_1^3 x_2^2 - x_1.$$

The matrix of the linearization at the point $(1, 1)$ is

(a) $\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$.

(b) $\begin{pmatrix} -1 & 1 \\ 3 & 2 \end{pmatrix}$.

(c) $\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}$.

(d) $\begin{pmatrix} -1 & 1 \\ 2 & 2 \end{pmatrix}$.

2. A water tank initially contains 20 liters of water, in which 100 grams of salt are dissolved. Water containing 5 grams of salt per liter enters the tank at a rate of 3 liters per minute, and the well mixed solution leaves the tank at a rate of 4 liters per minute. Let $Q(t)$ be the amount of salt in the tank. If Q is measured in grams and time in minutes, then the differential equation for $Q(t)$ is

(a) $Q'(t) = 15 - Q(t)/5$, $Q(0) = 100$.

(b) $Q'(t) = 15 - 4Q(t)/(20 - t)$, $Q(0) = 100$.

(c) $Q'(t) = 5 - 4Q(t)$, $Q(0) = 100$.

(d) $Q'(t) = 15t - Q(t)/5$, $Q(0) = 100$.

3. For the system

$$x' = -x + 3y,$$
$$y' = 2x - y,$$

the origin is a

(a) stable node.
(b) stable focus.
(c) saddle.
(d) unstable focus.

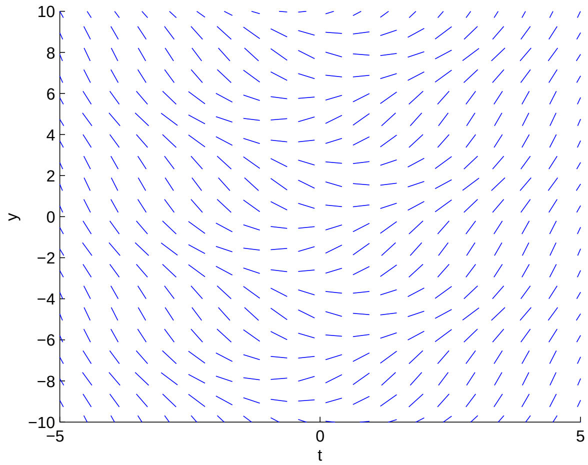
4. You solve the initial value problem $y' = 3y + t$, $y(1) = 1$, using the Euler method with $h = 0.1$. Then the approximation you find for $y(1.2)$ is

(a) 2.12.
(b) 1.8.
(c) 1.93.
(d) 1.4.

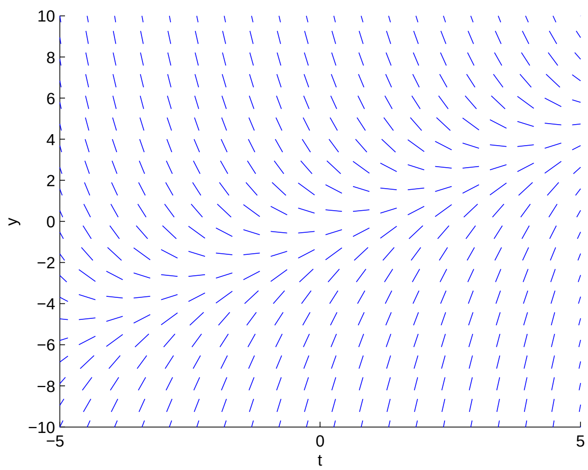
5. A particular solution of the equation $y''' + y = e^t + e^{-t}$ should have the form

(a) $ate^{-t} + be^t$.
(b) $ate^{-t} + bte^t$.
(c) $ae^{-t} + be^t$.
(d) $at^3e^{-t} + be^t$.

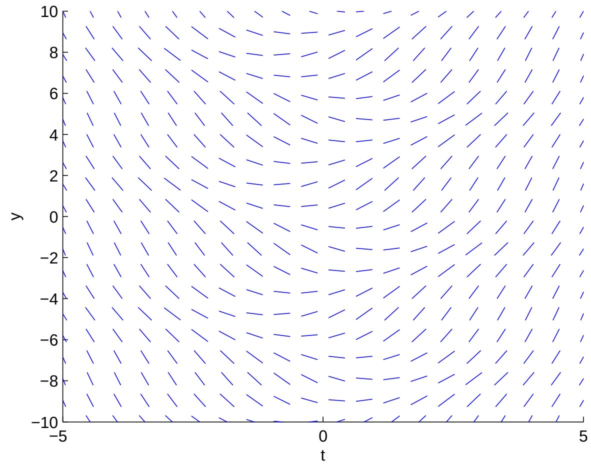
6. Which of the following direction fields is for the equation $y' = t - \sin(y)$?



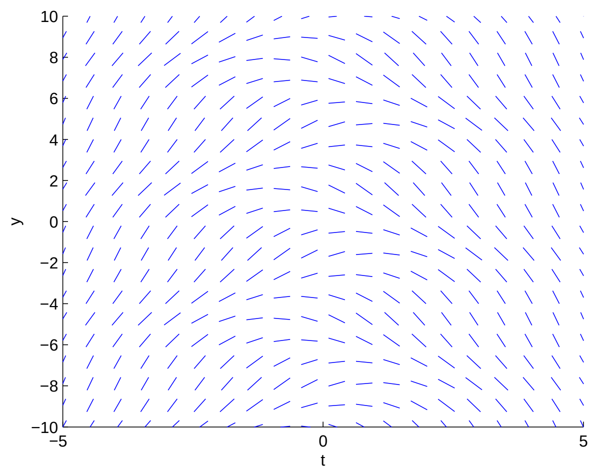
(a)



(b)



(c)



(d)

7. Consider the system

$$x' = -x + y,$$
$$y' = -y - x,$$

with initial condition $x(0) = 1$, $y(0) = 2$. Then $x(1)$ is

(a) $2e^2 - 1$.

(b) $3e^3 - 2e$.

(c) $(\cos(1) + 2\sin(1))/e$.

(d) $(2\cos(1) - \sin(1))/e$.

8. If $x' = x^2 + 4$, and $x(0) = 2\tan(1)$, then $x(0.1)$ is

(a) $2.1\tan(1)$.

(b) $2\tan(1.1)$.

(c) $2\tan(1.2)$.

(d) $2\tan(1.4)$.

9. The solution of the initial value problem

$$t(1-t)y' = (t-3)y + \frac{1}{\sin(4-t)}, \quad y(2) = 5,$$

is guaranteed to exist on the interval

(a) $(4 - 2\pi, 4)$.

(b) $(1, 4)$.

(c) $(4 - \pi, 3)$.

(d) $(4 - \pi, 4)$.

10. For $t \to \infty$, the solutions of the initial value problem $y' = y^2 - 9$, $y(0) = 2$ will

(a) converge to -3.

(b) always converge to 3.

(c) converge to 0.

(d) become infinite in finite time.

11. The function $y = \sin t$ is not a solution of one of the following equations. Identify which it is.

(a) $y^{(4)} + y = 0$.

(b) $y'' - y = -2\sin t$.

(c) $y''' + y' = 0$.

(d) $y^{(4)} - y = 0$.

12. The general solution of the system $\mathbf{y}' = A\mathbf{y}$, where

$$A = \begin{pmatrix} 2 & 2 \\ 0 & 2 \end{pmatrix},$$

is

(a) $c_1 t e^{2t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

(b) $c_1 e^{2t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} 2t \\ 1 \end{pmatrix}$.

(c) $c_1 e^{2t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

(d) $c_1 e^{2t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.