Math 2214, Fall 2015, Form A

1. The general solution of the equation

$$y'' + y = \ln t$$

can be found using the form

(a) $u(t)\cos t + v(t)\sin t + w(t)\ln t$.

(b) $u(t)e^{t} + v(t)e^{-t} + w(t)\ln t$.

(c) $u(t)\cos t + v(t)\sin t$.

(d) $u(t)e^{t} + v(t)e^{-t}$.

2. The solution of the initial value problem

$$\cos 2t\; y'' + t^2 y' = \ln t, \qquad y(\pi/2) = 2, \qquad y'(\pi/2) = -1,$$

is guaranteed to exist on the interval

(a) $\pi/4 < t < 3\pi/4$.

(b) $-\pi/4 < t < \pi/4$.

(c) $-\pi/4 < t < 3\pi/4$.

(d) $t > 0$.

3. The solution of the initial value problem

$$y'' + \alpha y' + \beta y = -4\cos 4t - 18\sin 4t,\; y(0) = y_0,\; y'(0) = y_0',$$

is $y(t) = e^{-t} + 2e^{2t} + \sin 4t$. Then, α, β, y_0 and y_0' are given by

(a) $\alpha = 1$, $\beta = -2$, $y_0 = 3$, and $y_0' = 7$.

(b) $\alpha = 1$, $\beta = 2$, $y_0 = 3$, and $y_0' = 9$.

(c) $\alpha = -1$, $\beta = 1$, $y_0 = 3$, and $y_0' = 3$.

(d) $\alpha = -1$, $\beta = -2$, $y_0 = 3$, and $y_0' = 7$.

4. What is the integrating factor for the equation

$$y' + \frac{1}{t^2+1} y = \tan(t)?$$

   (a) $\mu(t) = e^{\arctan(t)}$.
   (b) $\mu(t) = e^{\tan(t)}$.
   (c) $\mu(t) = e^{\frac{t}{t^2+1}}$.
   (d) $\mu(t) = e^{-(t^2+1)}$.

5. Suggest a trial form for the particular solution to the following equation

$$y'' + 4y' + 3y = t^2 e^{3t} + \cos(3t).$$

   (a) $y_P(t) = te^{3t}(At^2 + Bt + C) + D\cos(3t)$.
   (b) $y_P(t) = e^{3t}At^2 + B\sin(3t) + C\cos(3t)$.
   (c) $y_P(t) = te^{3t}(At^2 + Bt + C) + D\sin(3t) + E\cos(3t)$.
   (d) $y_P(t) = e^{3t}(At^2 + Bt + C) + D\sin(3t) + E\cos(3t)$.

6. You use the Euler method with a step size of 0.1 to solve the initial value problem

$$x' = x, \qquad x(0) = 2.$$

   Then your approximation for $x(0.3)$ is

   (a) 2.6.
   (b) 2.662.
   (c) 2.699.
   (d) 2.653.

7. A 2×2 matrix A has a double eigenvalue $\lambda = 2$. If the matrix has only one eigenvector $\bar{x} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and one generalized eigenvector $\bar{v} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$, then the general solution has the form

(a) $\bar{y}(t) = c_1 \begin{bmatrix} 2e^{2t} \\ e^{2t} \end{bmatrix} + c_2 \begin{bmatrix} (2+2t)e^{2t} \\ (1-3t)e^{2t} \end{bmatrix}.$

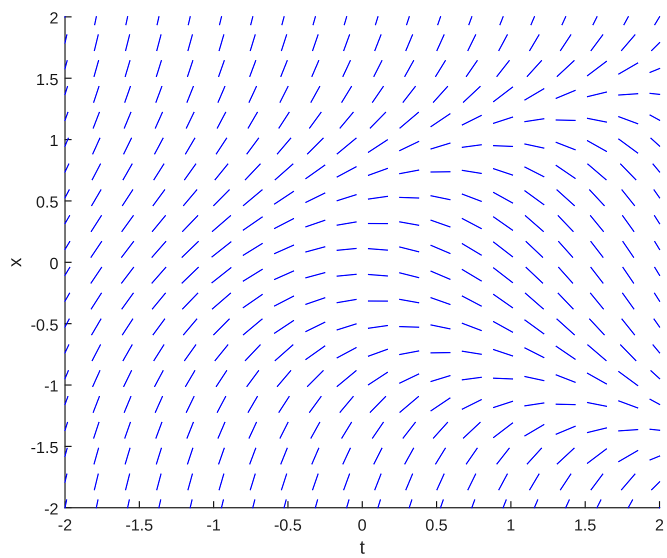
(b) $\bar{y}(t) = c_1 \begin{bmatrix} 2e^{2t} \\ e^{2t} \end{bmatrix} + c_2 \begin{bmatrix} (2t+2)e^{2t} \\ (t-3)e^{2t} \end{bmatrix}.$

(c) $\bar{y}(t) = c_1 \begin{bmatrix} 2e^{2t} \\ e^{2t} \end{bmatrix} + c_2 \begin{bmatrix} 2te^{2t} \\ te^{2t} \end{bmatrix} + c_3 \begin{bmatrix} 2e^{2t} \\ -3e^{2t} \end{bmatrix}.$

(d) $\bar{y}(t) = c_1 \begin{bmatrix} 2e^{2t} \\ e^{2t} \end{bmatrix} + c_2 \begin{bmatrix} 2e^{2t} \\ e^{2t} \end{bmatrix} + c_3 \begin{bmatrix} 2te^{2t} \\ -3te^{2t} \end{bmatrix}.$

8. The following figure shows a direction field. This direction field is for the equation

   (a) $x' = t^2 - x$.
   (b) $x' = t - x^2$.
   (c) $x' = x^2 - t$.
   (d) $x' = x - t^2$.



9. The system

$$x' = -x + y, \qquad y' = x(x + y - 2)$$

   has a stationary point at $x = y = 1$. This point is a

   (a) center.
   (b) saddle.
   (c) node.
   (d) focus.

10. Consider the equation

$$t^2y'' - ty' - 3y = 0 \quad t > 0.$$

Which of the following functions solve the equation?

I: $\frac{1}{t}$ **II:** t **III:** t^2 **IV:** t^3

(a) **II** and **III**.
(b) **II** and **IV**.
(c) **I** and **III**.
(d) **I** and **IV**.

11. A forced oscillation is described by the equation

$$u'' + 0.2u' + 25u = \cos(\omega t).$$

You should expect particularly large oscillations when ω is

(a) close to 25.
(b) small.
(c) large.
(d) close to 5.

12. The general solution of the system $\mathbf{y}' = A\mathbf{y}$, where

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},$$

is

(a) $c_1e^t\begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2e^{-t}\begin{pmatrix} -1 \\ 1 \end{pmatrix}$.
(b) $c_1\begin{pmatrix} \cos t \\ -\sin t \end{pmatrix} + c_2\begin{pmatrix} \sin t \\ \cos t \end{pmatrix}$.
(c) $c_1e^t\begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2e^{-t}\begin{pmatrix} -1 \\ -1 \end{pmatrix}$.
(d) $c_1\begin{pmatrix} \cos t \\ \sin t \end{pmatrix} + c_2\begin{pmatrix} \sin t \\ \cos t \end{pmatrix}$.