Test Version A

Instructions: Fill in **A** in the Test Version section. Then enter your NAME, ID Number, CRN (under Class ID) and write **A** (under Test ID) on the op-scan sheet. Darken the appropriate circles below your ID number and Class ID (CRN). **Use a number 2 pencil**. Machine grading may ignore faintly marked circles.

Mark your answers to the test questions in rows 1–15 of the op-scan sheet. Your score on this test will be the number of correct answers. You have one hour to complete this portion of the exam. Turn in the op-scan sheet with your answers, this exam, and all scrap paper at the end of this part of the final exam.

Exam Policies: You may not use a book, notes, formula sheet, or a calculator or computer. Giving or receiving unauthorized aid is an Honor Code Violation.

Signature: ______________________________

Name (printed): ___________________________

Student ID #: _____________________________

1. Which set is a basis for the null space of $\begin{bmatrix} 1 & 0 & 3 \\ 2 & -3 & 0 \\ -1 & 2 & 1 \end{bmatrix}$?

(A) $\left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ -3 \\ 2 \end{bmatrix} \right\}$

(B) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \right\}$

(C) $\left\{ \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix} \right\}$

(D) $\left\{ \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} \right\}$

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2. Which matrix corresponds to a linear transformation with range equal to $\mathbb{R}^2$?

(A) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 4 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \end{bmatrix}$

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3. Let $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}$ be linearly independent vectors in $\mathbb{R}^3$. Which statement is **false**?

(A) The vector $\boldsymbol{u} + \boldsymbol{v} + \mathbf{2}\boldsymbol{w}$ is in $\text{span}(\boldsymbol{u} + \boldsymbol{v}, \boldsymbol{w})$.

(B) The zero vector is in $\text{span}(\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w})$.

(C) The vectors $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}$ span $\mathbb{R}^3$.

(D) The vector $\boldsymbol{w}$ is in $\text{span}(\boldsymbol{u}, \boldsymbol{v})$.

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4. Which statement is **true** for any matrix A?

   I. If $\text{rank}(A)$ is equal to the number of columns of A, then the linear system $A\boldsymbol{x} = \boldsymbol{b}$ has a solution for all $\boldsymbol{b}$.

   II. If $\text{rank}(A)$ is equal to the number of rows of A, then the linear system $A\boldsymbol{x} = \boldsymbol{b}$ has a solution for all $\boldsymbol{b}$.

   (A) Only I.

   (B) Only II.

   (C) Both I and II.

   (D) Neither I nor II.

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5. If $\boldsymbol{u}$ and $\boldsymbol{v}$ are vectors such that

$$\|\boldsymbol{u}\| = 3, \qquad \|\boldsymbol{v}\| = 2, \qquad \boldsymbol{u} \cdot \boldsymbol{v} = -1$$

   then $(\mathbf{u} + 2\mathbf{v}) \cdot (\mathbf{u} + 2\mathbf{v})$ equals

   (A) 7.

   (B) 11.

   (C) 18.

   (D) 21.

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6. Let $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. Which statement is **true**?

   (A) Only A is diagonalizable.

   (B) Only B is diagonalizable.

   (C) Both A and B are diagonalizable.

   (D) Neither A nor B is diagonalizable.

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7. Which set is a basis for the eigenspace of $\begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 2 \\ 2 & 0 & 1 \end{bmatrix}$ corresponding to eigenvalue $\lambda = -1$?

(A) $\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$

(B) $\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \right\}$

(C) $\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

(D) $\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

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8. Let a be a real number. The linear system $A\boldsymbol{x} = \boldsymbol{b}$ with

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 5 & 4 & 3 & 2 \\ 7 & 7 & 7 & 7 & 7 \end{bmatrix}, \qquad \boldsymbol{b} = \begin{bmatrix} 0 \\ 0 \\ a \end{bmatrix}$$

has a solution

(A) only if $a = 0$.

(B) only if $a = 7$.

(C) for all values of a.

(D) for no value of a.

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9. If A is a 7×11 matrix such that $\operatorname{rank}(A) = 5$, then $\operatorname{nullity}(A^T)$ is equal to

(A) 2.

(B) 5.

(C) 6.

(D) Insufficient information to determine $\operatorname{nullity}(A^T)$.

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10. Which statement is **true**?

(A) The matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is a basis for $\mathbb{R}^2$.

(B) The set $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^2$.

(C) The set $\left\{ \begin{bmatrix} 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^2$.

(D) A set of any two vectors in $\mathbb{R}^2$ that spans $\mathbb{R}^2$ is a basis for $\mathbb{R}^2$.

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11. Find the standard matrix of the linear map that acts on the xy-plane by a 90° clockwise rotation followed by reflection about the line $y = x$.

(A) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(B) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

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12. Which one of the following is a steady-state vector for the transition matrix $\begin{bmatrix} .6 & .3 \\ .4 & .7 \end{bmatrix}$?

(A) $\begin{bmatrix} 3/4 \\ 1 \end{bmatrix}$

(B) $\begin{bmatrix} 3 \\ 1/4 \end{bmatrix}$

(C) $\begin{bmatrix} -3/4 \\ 1 \end{bmatrix}$

(D) $\begin{bmatrix} 3/4 \\ 1/4 \end{bmatrix}$

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13. Let a be a real number. The matrix $\begin{bmatrix} 2 & 2 & 2 \\ 3 & 1 & a \\ 1 & 3 & 0 \end{bmatrix}$ is invertible

(A) for all but one value of a.

(B) for exactly one value of a.

(C) for all values of a.

(D) for no value of a.

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14. Which set is an **orthogonal basis** for the column space of $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix}$?

(A) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

(B) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \right\}$

(C) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \right\}$

(D) None of the above.

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15. Which set is **not** a subspace of $\mathbb{R}^3$?

(A) null $\left(\begin{bmatrix} 0 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \right)$

(B) span $\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \right)$

(C) $\left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \right\}$

(D) All of the given sets are subspaces of $\mathbb{R}^3$.
