Test Version A

Instructions: Fill in **A** in the Test Version section. Then enter your NAME, ID Number, CRN (under Class ID) and write **A** (under Test ID) on the op-scan sheet. Darken the appropriate circles below your ID number and Class ID (CRN). **Use a number 2 pencil**. Machine grading may ignore faintly marked circles.

Mark your answers to the test questions in rows 1–14 of the op-scan sheet. Your score on this test will be the number of correct answers. You have one hour to complete this portion of the exam. Turn in the op-scan sheet with your answers, this exam, and all scrap paper at the end of this part of the final exam.

Exam Policies: You may not use a book, notes, formula sheet, or a calculator or computer. Giving or receiving unauthorized aid is an Honor Code Violation.

Signature: ________________________________

Name (printed): ____________________________

Student ID #: ______________________________

1. Let $\mathbf{v}_1, \mathbf{v}_2 \in \mathbb{R}^n$. Let S be the set of all vectors in $\mathbb{R}^n$ that are orthogonal to both $\mathbf{v}_1$ and $\mathbf{v}_2$. That is,

$$S = \{\mathbf{x} \in \mathbb{R}^n : \mathbf{x} \cdot \mathbf{v}_1 = 0 \text{ and } \mathbf{x} \cdot \mathbf{v}_2 = 0\}.$$

Which of the following statements is TRUE?

(A) S is a subspace of $\mathbb{R}^n$.

(B) S is not a subspace of $\mathbb{R}^n$ because $\mathbf{0} \notin S$.

(C) S is not a subspace of $\mathbb{R}^n$ because S is not closed under vector addition.

(D) S is not a subspace of $\mathbb{R}^n$ because S is not closed under scalar multiplication.

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2. Suppose that A is *row equivalent* to $\begin{bmatrix} 1 & 0 & -2 & 3 & 0 & -4 \\ 0 & 1 & -2 & 2 & 0 & -7 \\ 0 & 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$. Which of the following is a basis for null(A)?

(A) $\left\{ \begin{bmatrix} -2 \\ -2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ -7 \\ 4 \\ 0 \end{bmatrix} \right\}$

(B) $\left\{ \begin{bmatrix} 1 \\ 0 \\ -2 \\ 3 \\ 0 \\ -4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -2 \\ 2 \\ 0 \\ -7 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 4 \end{bmatrix} \right\}$

(C) $\left\{ \begin{bmatrix} 2 \\ 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ -2 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 7 \\ 0 \\ 0 \\ -4 \\ 1 \end{bmatrix} \right\}$

(D) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

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3. Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation defined by $T(\mathbf{x}) = A\mathbf{x}$ where $A = \begin{bmatrix} 1 & -5 & -3 \\ -3 & 15 & 9 \end{bmatrix}$. If $\begin{bmatrix} 2 \\ b \end{bmatrix}$ is in the range of T, find the value of b.

(A) 0

(B) -6

(C) There is more than one possible value for b.

(D) There is not enough information to compute b.

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4. Find the rank of the following matrix A:

$$A = \begin{bmatrix} 1 & -4 & 9 & -7 \\ -1 & 2 & -4 & 1 \\ 5 & -6 & 10 & 7 \end{bmatrix}$$

(A) $\text{rank}(A) = 1$ (B) $\text{rank}(A) = 2$ (C) $\text{rank}(A) = 3$ (D) $\text{rank}(A) = 4$

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5. Let A be an $n \times n$ matrix. Suppose that there exists an $n \times n$ matrix B such that $AB = I_n$ and $BA = I_n$. Which of the following statements are TRUE?

(i) The equation $A\mathbf{x} = \mathbf{b}$ has a unique solution for every $\mathbf{b} \in \mathbb{R}^n$.

(ii) The transformation $\mathbf{x} \mapsto A\mathbf{x}$ is not onto.

(iii) The equation $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution.

(iv) The columns of A span $\mathbb{R}^n$.

(v) A is invertible.

(A) (ii) and (iii) only

(B) (iv) and (v) only

(C) (i), (iv), and (v) only

(D) (i), (ii), (iv), and (v) only

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6. Which of the following linear systems has a *unique* solution?

(A) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 9 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

(C) Both (A) and (B).

(D) Neither (A) nor (B).

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7. Find the determinant of the matrix A given by

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & 0 \\ x & y & -1 \end{bmatrix}$$

(A) $1 + 2x + 3y$ (B) $1 - 2x - 3y$ (C) $1 + 2x - 3y$ (D) $1 - 2x + 3y$

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8. Consider the matrix $A = \begin{bmatrix} -1 & \frac{1}{2} \\ 3 & -\frac{1}{2} \end{bmatrix}$ and vector $\mathbf{v} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$.

Define

$$\begin{bmatrix} x_k \\ y_k \end{bmatrix} = A^k \mathbf{v}.$$

Which of the following describes the behavior of x_k as k grows large?

(A) x_k approaches 0.

(B) x_k approaches a nonzero finite value.

(C) x_k becomes negative and large in magnitude.

(D) x_k grows large in magnitude and alternates in sign.

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9. Suppose that A is an invertible $n \times n$ matrix and $\mathbf{b}$ is a nonzero vector in $\mathbb{R}^n$. Consider the following statements:

(i) $\mathbf{b}$ is in col(A).

(ii) $\mathbf{b}$ is in null(A).

Which of the above statements are TRUE?

(A) (i) only

(B) (ii) only

(C) Both (i) and (ii)

(D) Neither (i) nor (ii)

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10. S is a set of vectors in $\mathbb{R}^3$ that are linearly independent, but do not span $\mathbb{R}^3$. What is the maximum number of vectors in S?

(A) one

(B) two

(C) three

(D) S may contain any number of vectors.

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11. If A and B are two invertible $n \times n$ square matrices, then

(A) AB is also invertible and $(AB)^{-1} = A^{-1}B^{-1}$.

(B) AB is also invertible and $(AB)^{-1} = B^{-1}A^{-1}$.

(C) $A + B$ is also invertible and $(A+B)^{-1} = A^{-1} + B^{-1}$.

(D) $A + B$ is also invertible and $(A+B)^{-1} = A^{-1} - B^{-1}$.

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12. Let A be an $n \times n$ square matrix with exactly three distinct eigenvalues and the dimension of each of its eigenspaces is 2 or less. Given that A is diagonalizable, find the value of n.

(A) $3 \leq n \leq 6$

(B) $n < 3$

(C) $n > 6$

(D) There is not enough information to estimate the value of n.

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13. Find the *orthogonal projection* of $\mathbf{v}$ onto the subspace $W = \text{span}(\mathbf{u}_1, \mathbf{u}_2)$ where

$$\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \qquad \mathbf{u}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \qquad \mathbf{u}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}.$$

(A) $\text{proj}_W(\mathbf{v}) = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$

(B) $\text{proj}_W(\mathbf{v}) = \begin{bmatrix} 2 \\ 0 \\ 6 \end{bmatrix}$

(C) $\text{proj}_W(\mathbf{v}) = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$

(D) $\text{proj}_W(\mathbf{v}) = \begin{bmatrix} -1/2 \\ 0 \\ 3/2 \end{bmatrix}$

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14. Let A and B be two $n \times n$ square matrices. Suppose that the first column of AB is entirely zero, but the first column of B is not entirely zero. Then

(A) $\det(A) \neq 0$.

(B) $\det(A)$ cannot be computed with the given information.

(C) The columns of A are linearly dependent.

(D) There is not enough information on columns of A.
