Test Version A

Instructions: Fill in A in the Test Version section. Then enter your NAME, ID Number, CRN (under Class ID) and write A (under Test ID) on the op-scan sheet. Darken the appropriate circles below your ID number and Class ID (CRN). **Use a number 2 pencil**. Machine grading may ignore faintly marked circles.

Mark your answers to the test questions in rows 1–15 of the op-scan sheet. Your score on this test will be the number of correct answers. You have one hour to complete this portion of the exam. Turn in the op-scan sheet with your answers, this exam, and all scrap paper at the end of this part of the final exam.

Exam Policies: You may not use a book, notes, formula sheet, or a calculator or computer. Giving or receiving unauthorized aid is an Honor Code Violation.

Signature: ______________________________

Name (printed): ______________________________

Student ID #: ______________________________

1. Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation defined by

$$T(x_1, x_2, x_3) = (5x_1 - 3x_3, 2x_1 - x_2 + x_3).$$

Which of the following is the standard matrix for the transformation T?

(A) $\begin{bmatrix} -3 & 1 \\ 0 & -1 \\ 5 & 2 \end{bmatrix}$

(B) $\begin{bmatrix} 2 & -1 & 1 \\ 5 & 0 & -3 \end{bmatrix}$

(C) $\begin{bmatrix} 5 & 2 \\ 0 & -1 \\ -3 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 5 & 0 & -3 \\ 2 & -1 & 1 \end{bmatrix}$

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2. Consider the subset, W, of $\mathbb{R}^2$ defined as follows:

$$W = \left\{ \begin{bmatrix} a \\ b \end{bmatrix} \in \mathbb{R}^2 : \ 2a - 3b = 0 \right\}.$$

Is W a subspace of the vector space $\mathbb{R}^2$?

(A) W is not a subspace, because it is not closed under addition.

(B) W is not a subspace, because it contains only the zero vector.

(C) W is a subspace, because it contains the zero vector.

(D) W is a subspace, because it is a null-space for a 1×2 matrix.

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3. Let $A = \begin{bmatrix} \frac{5}{4} & \frac{3}{4} \\ 0 & \frac{1}{2} \end{bmatrix}$, and let $\mathbf{x}_{i+1} = A\mathbf{x}_i$ be a discrete dynamical system. Which statement is true?

(A) The origin is an attractor.

(B) The origin is a repeller.

(C) The origin is a saddlepoint.

(D) None of the above.

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4. Suppose A is a 7×5 matrix. The echelon form of this matrix has 4 pivots. Which pair of statements below is correct?

(A) $\dim \text{Nul} A = 1$, $\dim \text{Nul} A^T = 3$

(B) $\dim \text{Nul} A = 1$, $\dim \text{Col} A = 7$

(C) $\dim \text{Col} A = 4$, $\dim \text{Nul} A = 3$

(D) $\text{rank} A^T = 5$, $\dim \text{Nul} A^T = 2$

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5. Given $A = PDP^{-1}$, where

$$A = \begin{bmatrix} 2 & -3 \\ 2 & -3 \end{bmatrix} \quad P = \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} \quad D = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix},$$

find the value of A^{100}.

(A) $\begin{bmatrix} -2^{100} & -3 \\ 2 & 0 \end{bmatrix}$

(B) $\begin{bmatrix} -2 & 3 \\ -2 & 3 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 3^{100} \\ 1 & 2^{100} \end{bmatrix}$

(D) $\begin{bmatrix} 2^{100} & 0 \\ 0 & 0 \end{bmatrix}$

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6. Let

$$A = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 12 \\ -4 & h \end{bmatrix}.$$

For what values of h, if any, does $AB = BA$?

(A) $h = -48$

(B) $h = -3$

(C) No value of h

(D) Any value of h

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7. If the matrix A has characteristic equation $\lambda^3 - \lambda = 0$, which of the following is **not true**?

(A) A is invertible.

(B) A is diagonalizable.

(C) A is 3×3.

(D) A^T has characteristic equation $\lambda^3 - \lambda = 0$.

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8. Let

$$A = \begin{bmatrix} 1 & -2 & 3 & 5 \\ 0 & 2 & -4 & 4 \\ 2 & 0 & 2 & 14 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Consider the following statements.

1. Every column of A is a pivot column.
2. For every $\mathbf{b} \in \mathbb{R}^4$, the matrix equation $A\mathbf{x} = \mathbf{b}$ has a solution.
3. Every vector in $\mathbb{R}^4$ can be written as a linear combination of the columns of A.
4. There is some vector $\mathbf{b} \in \mathbb{R}^4$ such that the matrix equation $A\mathbf{x} = \mathbf{b}$ has an infinite number of solutions.

Which of the following is true?

(A) Only 1, 2, and 3 are true.

(B) Only 2 is true.

(C) Only 2 and 3 are true.

(D) Only 3 and 4 are true.

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9. $\mathbf{u}$ is a solution to the system $A\mathbf{x} = \mathbf{b}$ and $\mathbf{v}$ is a solution to the system $A\mathbf{x} = \mathbf{0}$. Which of the following is a solution to $A\mathbf{x} = 2\mathbf{b}$?

(A) $2\mathbf{v} + \mathbf{u}$

(B) $4\mathbf{u} - 2\mathbf{v}$

(C) $2\mathbf{u} + 3\mathbf{v}$

(D) None of the above

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10. It is given that

$$\det \begin{bmatrix} a & b & c \\ e & f & g \\ h & i & j \end{bmatrix} = 24.$$

Compute

$$\det \begin{bmatrix} 2a & 2b & 2c \\ e-2a & f-2b & g-2c \\ 2a-h & 2b-i & 2c-j \end{bmatrix}.$$

(A) 48

(B) -48

(C) 12

(D) -12

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11. A is a 5×6 matrix; B is row-equivalent to A and B is in reduced echelon form. You do not know all of the entries in B, but you do know that the third row of B is $[0\ 0\ 0\ 0\ 1\ 0]$. What are the only possible values for the rank of A?

(A) 1, 2, 3, 4, 5

(B) 2, 3

(C) 3, 4

(D) 3, 4, 5

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12. The vectors $\mathbf{u}$ and $\mathbf{v}$ lie in $\mathbb{R}^3$, and you know that $\mathbf{u}$ and $\mathbf{v}$ are orthogonal, $\|\mathbf{u}\| = 2$, and $\|\mathbf{v}\| = 3$. Find $(2\mathbf{u} - \mathbf{v}) \cdot 2\mathbf{v}$.

(A) There is not enough information to know.

(B) -18

(C) 0

(D) 6

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13. Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$, where $\mathbf{b}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\mathbf{b}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. If $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, then $\mathbf{x} =$

(A) $\begin{bmatrix} -2 \\ 5 \end{bmatrix}$.

(B) $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$.

(C) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

(D) $\begin{bmatrix} 5 \\ 3 \end{bmatrix}$.

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14. Which option describes a solution set of the following system of linear equations?

$$\begin{array}{rcrcrcrcr} x_1 & - & 2x_2 & + & x_3 & & & = & 11 \\ & & & & x_3 & - & 3x_4 & = & -7 \\ x_1 & - & 2x_2 & + & x_3 & + & x_4 & = & 13 \end{array}$$

(A) $x_1 = 12$, $x_3 = -1$, $x_4 = 2$.

(B) $x_1 = 12 + 2x_2$, x_2 is free, $x_3 = -9$, $x_4 = 2$.

(C) $x_1 = 12 + 2x_2$, x_2 is free, $x_3 = -1$, $x_4 = 2$.

(D) There is no solution.

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15. A 4×6 augmented matrix represents a linear system, $A\mathbf{x} = \mathbf{y}$, with ...

(A) 4 equations in 5 variables.

(B) 4 equations in 6 variables.

(C) 6 equations in 3 variables.

(D) 6 equations in 4 variables.
