

## Test Version: A

**INSTRUCTIONS:** Please enter your **NAME**, **ID Number**, **Test Version**, and your **CRN** on the opscan sheet. The CRN should be written in the field labeled **Class ID** and the test version (A, B, or C) under **Test ID**. Leave the Date, Instructor/Class, Test Name, and Time fields blank. Darken the appropriate circles below your ID number, below Class ID, and beside Test Version. **Use a number 2 pencil.** Machine grading may ignore faintly marked circles.

Mark your answers to the test questions in rows 1-13 of the opscan sheet. Your score on this test will be the number of correct answers. You have one hour to complete this portion of the exam. Turn in the opscan sheet with your answers and the question sheets at the end of this part of the final exam.

**Exam Policies:** You may not use a book, notes, formula sheet, calculator or a computer. Giving or receiving unauthorized aid is an Honor Code Violation.

Signature: \_\_\_\_\_

Name (printed): \_\_\_\_\_

Student ID: # \_\_\_\_\_

[1] Evaluate  $\int_0^1 \int_{2y}^2 \cos(x^2) \, dx \, dy$ .

- A)  $\sin(4) - \cos(4) + 1$       B)  $\frac{\sin(4)}{4}$       C)  $\sin(1)$       D)  $\sin(4) + \cos(4) - 1$

[2] Which of the following is an equation for the plane through the point  $(1, 1, -1)$  and parallel to the plane  $x - y + z = 3$ ?

- A)  $x + y - z = -1$       B)  $x + y - z = 3$       C)  $-x + y - z = 1$       D)  $x - y + z = -3$

[3] Evaluate  $\int_0^{\sqrt{2}} \int_0^x \sqrt{x^2 + y^2} \, dy \, dx + \int_{\sqrt{2}}^2 \int_0^{\sqrt{4-x^2}} \sqrt{x^2 + y^2} \, dy \, dx$ .

- A)  $\frac{2\pi}{3}$       B)  $\frac{\pi}{2}$       C)  $\frac{\sqrt{2}\pi}{6}$       D)  $\pi$

[4] Let  $f(x, y) = \frac{x^2 y}{x^4 + y^2}$ . Which of the following statements is **true** about  $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ ?

- A)  $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$  does **not** exist because  $\lim_{x \rightarrow 0} f(x, 0)$  does not exist.
- B)  $\lim_{(x,y) \rightarrow (0,0)} f(x, y) = 0$  because  $\lim_{x \rightarrow 0} f(x, kx) = 0$  for every  $k$ .
- C)  $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$  does **not** exist because  $\lim_{x \rightarrow 0} f(x, 0)$  is not equal to  $\lim_{x \rightarrow 0} f(x, x^2)$ .
- D)  $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$  does **not** exist because  $f(x, y)$  is undefined at  $(0, 0)$ .

[5] Find the unit tangent vector  $\mathbf{T}(t)$  to the curve  $\mathbf{r}(t) = \langle \sin t, 1 + t, \cos t \rangle$  when  $t = 0$ .

- A)  $\left\langle 0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$       B)  $\langle 0, 0, -1 \rangle$       C)  $\left\langle \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right\rangle$       D)  $\left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right\rangle$

[6] A ball is thrown into the air with initial velocity  $\mathbf{v}(0) = 3\mathbf{i} + 8\mathbf{k}$ . The acceleration is given by  $\mathbf{a}(t) = 8\mathbf{j} - 16\mathbf{k}$ . How far away is the ball from its initial position at  $t = 1$ ?

- A)  $2\sqrt{3}$       B) 3      C)  $4\sqrt{5}$       D) 5

[7] Two right circular cylindrical storage tanks A and B have volume  $V = \pi r^2 h$  with height 25m and radius 5m. The **radius** of tank A is increased by a small amount while the **height** of tank B is increased by the same small amount. Which of the following statements is true?

- A) The volume of tank A increased approximately 10 times more than the volume of tank B.
- B) The volume of tank B increased approximately 10 times more than the volume of tank A.
- C) The volume of tank A increased approximately 5 times more than the volume of tank B.
- D) The volume of tank B increased approximately 5 times more than the volume of tank A.

[8] Rewrite  $\int_{-2}^2 \int_{y^2}^4 \int_0^{2-x/2} dz dx dy$  in  $dx dz dy$  order.

- A)  $\int_{-2}^2 \int_0^{2-y^2/2} \int_{y^2}^{4-2z} dx dz dy$
- B)  $\int_{-2}^2 \int_0^{2-y^2/2} \int_{y^2}^4 dx dz dy$
- C)  $\int_{-2}^2 \int_0^2 \int_{y^2}^{4-2z} dx dz dy$
- D)  $\int_{-2}^2 \int_0^2 \int_{y^2}^4 dx dz dy$

[9] Find the maximum rate of change of  $f(x, y) = x^2 - xe^{2y}$  at the point  $(2, 0)$ .

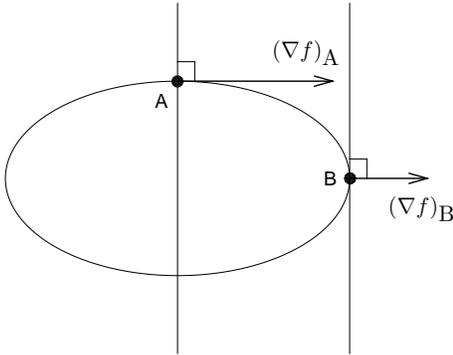
- A)  $\sqrt{6}$
- B) 3
- C) 5
- D) 6

[10] Suppose  $f(x, y)$  is a differentiable function of  $x$  and  $y$  and let  $g(r, s) = f(2rs, 8s - 2r)$ . Use the table of values to calculate  $g_r(2, 1)$ .

$(x, y)$	$f$	$f_x$	$f_y$
$(2, 1)$	2	-1	1
$(4, 4)$	3	2	3

- A) 0
- B) 1
- C) -4
- D) -2

- [11] In the figure below, straight lines represent level curves of a differentiable function  $f(x, y)$  and the ellipse represents the constraint  $g(x, y) = 0$ . The absolute maximum of  $f(x, y)$  subject to the constraint  $g(x, y) = 0$



- A) could occur at point B but not at point A.  
 B) could occur at point A but not at point B.  
 C) could occur at point A and could occur at point B.  
 D) could not occur at point A and could not occur at point B.

- [12] The absolute minimum of  $f(x, y) = x^2 + 4y^2 - 4y$  over the **line segment**  $x = 2$  with  $0 \leq y \leq 2$  equals

- A) 3      B) -1      C) 4      D) 0

- [13] Rewrite  $\int_0^{2\pi} \int_1^{\sqrt{2}} \int_{-\sqrt{2-r^2}}^{\sqrt{2-r^2}} r \, dz \, dr \, d\theta$  in spherical coordinates.

A)  $\int_0^{2\pi} \int_{\pi/4}^{3\pi/4} \int_{1/\cos \phi}^{\sqrt{2}} \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$

B)  $\int_0^{2\pi} \int_{\pi/4}^{3\pi/4} \int_{1/\sin \phi}^{\sqrt{2}} \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$

C)  $\int_0^{2\pi} \int_{\pi/4}^{3\pi/4} \int_0^{\sqrt{2}} \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$

D)  $\int_0^{2\pi} \int_{\pi/4}^{3\pi/4} \int_1^{\sqrt{2}} \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$