VirginiaTech

Math 2114

Common Final Exam
May 6, 2016
Form A

Instructions: Using a #2 pencil only, write your name and your instructor's name in the blanks provided. Write your student ID number and your CRN in the blanks marked "ID number" and "Class ID", and bubble in the appropriate spaces on the form.

Important: Mark your form letter **A** at the very top of your form next to the words "Test Version". Your form cannot be processed without this information.

You have one hour to complete this part of the final exam. Mark your answers to the test questions in rows 1–16 of the answer sheet. Your score on this part of the exam will be the number of correct answers. There is no penalty for guessing.

At the end of the hour, turn in both this test with the cover sheet intact, and the answer sheet.

Honor Policy: You may not use a book, notes, formula sheet, or any electronic device during this exam. Giving or receiving unauthorized aid is a violation of the undergraduate honor code.

Name: ______________________________

Signature: ______________________________

Student ID number: ______________________________

1. Let $H = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x = 0 \text{ or } y = 0 \right\}$. Which of the following statements is true?

   (a) H is a subspace of $\mathbb{R}^2$ because $c\mathbf{u}$ is in H for all scalars c and all $\mathbf{v}$ in H.

   (b) H is a subspace of $\mathbb{R}^2$ because H spans $\mathbb{R}^2$.

   (c) H is a subspace of $\mathbb{R}^2$ because the zero vector is in H.

   (d) H is not a subspace of $\mathbb{R}^2$ because $\mathbf{u} + \mathbf{v}$ is not in H for all $\mathbf{u}$ and $\mathbf{v}$ in H.

   (e) H is not a subspace of $\mathbb{R}^2$ because $c\mathbf{u}$ is not in H for all scalars c and all $\mathbf{v}$ in H.

2. Let

$$A = \begin{bmatrix} 1 & 2 & 1 & 3 \\ 2 & 4 & 4 & 5 \\ 1 & 2 & 3 & 2 \end{bmatrix}$$

   A basis for Col A is given by

   (a) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} \right\}$

   (b) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \right\}$

   (c) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} \right\}$

   (d) $\left\{ \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} \right\}$

   (e) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} \right\}$

3. Find the eigenvalues of $A = \begin{bmatrix} 3 & -5 \\ 9 & -3 \end{bmatrix}$

   (a) $\lambda = 4, \lambda = -9$

   (b) $\lambda = 3i, \lambda = -3i$

   (c) $\lambda = 6i, \lambda = -6i$

   (d) $\lambda = 3, \lambda = -3$

   (e) $\lambda = 11, \lambda = -3$

4. Let $\mathbf{u}, \mathbf{v}$ be vectors in $\mathbb{R}^n$ with θ the angle between $\mathbf{u}$ and $\mathbf{v}$. Then $\|\mathbf{u} + \mathbf{v}\|^2$ is equal to:

   (a) $\|\mathbf{u}\|^2 \|\mathbf{v}\|^2 \cos\theta$

   (b) $\|\mathbf{u}\|^2 + \|\mathbf{v}\|^2$

   (c) $\|\mathbf{u}\|^2 - \|\mathbf{v}\|^2$

   (d) $\|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 + 2\|\mathbf{u}\| \|\mathbf{v}\| \cos\theta$

   (e) $\|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 - 2\|\mathbf{u}\| \|\mathbf{v}\| \cos\theta$

5. Suppose the matrix $A = \begin{bmatrix} 1 & -1 & -1 & -1 \\ 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ is an echelon form of an augmented matrix.

Then the solution to the system represented by A is given by

(a) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}.$

(b) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}.$

(c) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}.$

(d) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}.$

(e) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} + x_4 \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}.$

6. Find the determinant of the matrix

$$A = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 3 & 1 & 4 \\ 0 & 0 & 2 & 3 & 4 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(a) -4

(b) -2

(c) 4

(d) 8

(e) 12

7. $T(\mathbf{x}) : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation which first rotates points 90° counterclockwise about the origin, and then reflects the plane across the y-axis. Find the standard matrix A such that $T(\mathbf{x}) = A\mathbf{x}$.

(a) $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(b) $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(c) $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

(d) $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(e) $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

8. Suppose $A = PB$ where $A = \begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix}$ and $P = \begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix}$. Find the matrix B.

   (a) $B = \begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix}$
   (b) $B = \begin{bmatrix} -5 & 5 \\ -3 & 3 \end{bmatrix}$
   (c) $B = \begin{bmatrix} 0 & -1 \\ 0 & -2 \end{bmatrix}$
   (d) $B = \begin{bmatrix} 5/3 & -5/3 \\ 1 & -1 \end{bmatrix}$
   (e) Not possible because $\det(A) = 0$

9. Let $\mathbf{u}$ be a nonzero vector in $\mathbb{R}^5$ and $\mathbf{v}$ be a nonzero vector in $\mathbb{R}^3$. What is the rank of the matrix $A = \mathbf{u}\mathbf{v}^T$?

   (a) rank$(A) = 4$
   (b) rank$(A) = 3$
   (c) rank$(A) = 2$
   (d) rank$(A) = 1$
   (e) rank$(A) = 0$

10. Let A be an $n \times n$ matrix, $T(\mathbf{x}) = A\mathbf{x}$, and $\mathbf{b}$ be a vector in $\mathbb{R}^n$. If $\det(A) = 0$, then which of the following statements is **true**?

   (a) The transformation T is one-to-one.
   (b) The equation $A\mathbf{x} = \mathbf{b}$ has a unique solution.
   (c) The equation $A\mathbf{x} = \mathbf{0}$ has a non-trivial solution.
   (d) The columns of A span $\mathbb{R}^n$.
   (e) The columns of A form a basis for $\mathbb{R}^n$.

11. Determine whether the statements are true or false.

   (i) If A is an $n \times n$ matrix and $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution, then the columns of A span $\mathbb{R}^n$.
   (ii) If B is invertible, then the row vectors of B are linearly independent.
   (iii) If C is row equivalent to an invertible matrix D, then C is invertible.

   (a) Statement (i) is false, and statements (ii) and (iii) are true.
   (b) Statements (i) and (iii) are false, and statement (ii) is true.
   (c) Statements (i) and (ii) are false, and statement (iii) is true.
   (d) Statement (ii) is false, and statements (i) and (iii) are true.
   (e) All three statements are true.

12. Let

$$A = \begin{bmatrix} 5 & -1 & 3 & -1 \\ 0 & 4 & h & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Find h so that the eigenspace corresponding to the eigenvalue $\lambda = 5$ is 2-dimensional.

(a) $h = 0$

(b) $h = 2$

(c) $h = 3$

(d) $h = 5$

(e) $h = 6$

13. Which of the following statements are **true** for all 3×3 matrices A and B?

$$\begin{array}{rl} \text{I.} & (A-B)^2 = A^2 - 2AB + B^2 \\ \text{II.} & (AB)^2 = A^2B^2 \\ \text{III.} & (AB)^{-1} = A^{-1}B^{-1} \end{array}$$

(a) I only.

(b) II only.

(c) III only.

(d) More than one statement.

(e) None of the statements.

14. Suppose the 2×2 matrix A has eigenvalues $\lambda_1 = 4$ and $\lambda_2 = 3$, with eigenvectors $\mathbf{v}_1$ and $\mathbf{v}_2$, respectively. If $\mathbf{u} = 5\mathbf{v}_1 + \mathbf{v}_2$, then $A^2\mathbf{u}$ is equal to

(a) $25\mathbf{v}_1 + \mathbf{v}_2$

(b) $25\mathbf{v}_1 + 3\mathbf{v}_2$

(c) $80\mathbf{v}_1 + 9\mathbf{v}_2$

(d) $100\mathbf{v}_1 + 3\mathbf{v}_2$

(e) $400\mathbf{v}_1 + 9\mathbf{v}_2$

15. An $n \times n$ matrix B has characteristic polynomial $p(\lambda) = -\lambda(\lambda - 3)^3(\lambda - 2)^2(\lambda + 1)$. Which of the following statements is **false**?

    (a) rank $B = 6$.

    (b) det $(B) = 0$.

    (c) det $(B^T B) = 0$.

    (d) B is invertible.

    (e) $n = 7$.

16. Which of the following statements is **false**?

    (a) Let $\mathcal{S} = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_m\}$ be a subset of a vector space V with $\dim(V) = n$. If $m > n$, then $\mathcal{S}$ is linearly dependent.

    (b) If A is an $m \times n$ matrix, then dim Nul $A = n$.

    (c) If $\mathcal{B}$ is a basis for some finite dimensional vector space W, then the change of coordinates matrix $P_{\mathcal{B}}$ is always invertible.

    (d) $\dim(\mathbb{R}^{17}) = 17$.

    (e) If $\mathcal{B}_1$ and $\mathcal{B}_2$ are both bases for the same vector space, then $\mathcal{B}_1$ and $\mathcal{B}_2$ have the same number of vectors.