VirginiaTech

Math 2114

Common Final Exam

May 13, 2015

Form A

Instructions: Using a #2 pencil only, write your name and your instructor's name in the blanks provided. Write your student ID number and your CRN in the blanks marked "ID number" and "Class ID", and bubble in the appropriate spaces on the form.

Important: Mark your form letter **A** at the very top of your form next to the words "Test Version". Your form cannot be processed without this information.

You have one hour to complete this part of the final exam. Mark your answers to the test questions in rows 1–16 of the answer sheet. Your score on this part of the exam will be the number of correct answers. There is no penalty for guessing.

At the end of the hour, turn in both this test with the cover sheet intact, and the answer sheet.

Honor Policy: You may not use a book, notes, formula sheet, or any electronic device during this exam. Giving or receiving unauthorized aid is a violation of the undergraduate honor code.

Name: ______________________________

Signature: ______________________________

Student ID number: ______________________________

1. Let

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\},$$

and let $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ be a vector in Span $\mathcal{B}$. To find $[\mathbf{x}]_{\mathcal{B}}$ we multiply $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ on the left by which of the following matrices?

(a) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & -1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(e) $\begin{bmatrix} 0 & 0 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$

2. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation that maps $\mathbf{u} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ to $T(\mathbf{u}) = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$ and maps $\mathbf{v} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$ to $T(\mathbf{v}) = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$. What is the image of $2\mathbf{u} + 4\mathbf{v}$?

(a) $\begin{bmatrix} 14 \\ 6 \end{bmatrix}$

(b) $\begin{bmatrix} 22 \\ 20 \end{bmatrix}$

(c) $\begin{bmatrix} 20 \\ 22 \end{bmatrix}$

(d) $\begin{bmatrix} 6 \\ 14 \end{bmatrix}$

(e) There is not enough information to find the image.

3. Let S be the set of all real values of a such that the linear system represented by the following augmented matrix has infinitely many solutions. How many values are in S?

$$\begin{bmatrix} 2 & 6 & 8 \\ 1 & a & 4 \end{bmatrix}$$

(a) zero
(b) one
(c) three
(d) infinitely many
(e) There is not enough information to answer this question.

4. If A is a 4×7 matrix, what is the smallest possible dimension of Nul A?

(a) 3

(b) 4

(c) 1

(d) 0

(e) None of the above.

5. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix}$. Which of the following vectors is orthogonal to the column space of A?

(a) $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$

(c) $\begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$

(d) $\begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$

(e) Since the second row is a multiple of the third row, Col A is undefined.

6. Suppose $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, $\mathbf{v}_4$ are vectors in a 3 dimensional vector space V. Which statement is **true**?

(a) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ must be linearly dependent.

(b) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ could be a basis for V.

(c) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ must be a basis for V

(d) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ must span V.

(e) $\{\mathbf{v}_1, \mathbf{v}_2\}$ must be linearly independent.

7. Let $W = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x \geq 0 \text{ and } y \geq 0 \right\}$. Determine whether W is a subspace of $\mathbb{R}^2$.

(a) W is not a subspace because it is not closed under scalar multiplication.

(b) W is not a subspace because it is not closed under vector addition.

(c) W is not a subspace because it does not contain the zero vector.

(d) W is a subspace because it is the null space of a matrix.

(e) W is a subspace because it is a span of a set of vectors.

8. Which of the following matrices is the reduced row echelon form of the matrix

$$A = \begin{bmatrix} 1 & 2 & 4 & 1 \\ 2 & 4 & 6 & 0 \\ 4 & 6 & 8 & 0 \end{bmatrix}?$$

(a) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

(e) none of the above

9. You are given that

$$\mathbf{x}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \mathbf{x}_3 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, P = \begin{bmatrix} \mathbf{x}_1 & \mathbf{x}_2 & \mathbf{x}_3 \end{bmatrix}, D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & -4 \end{bmatrix},$$

and $A = PDP^{-1}$. Then $A^2\mathbf{x}_2 =$

(a) $\begin{bmatrix} 25 \\ 0 \\ 50 \end{bmatrix}$.

(b) $\begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$.

(c) $\begin{bmatrix} 5 \\ 0 \\ 20 \end{bmatrix}$.

(d) $\begin{bmatrix} 25 \\ 0 \\ 100 \end{bmatrix}$.

(e) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

10. Suppose A is a 2×2 matrix which is *left-invertible*. That is, there is a 2×2 matrix C such that $CA = I$. Which of the following statements must be **true?**

(a) A cannot be a diagonal matrix.

(b) A is row-equivalent to the matrix $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$.

(c) C cannot be a diagonal matrix.

(d) C is also left-invertible.

(e) $AC = -CA$

11. Suppose A and B are $n \times n$ matrices, $\det A = 2$ and $\det B = \frac{1}{2}$. Which is the **first** false statement in the list below?

(a) A must be invertible.

(b) A and B must both be invertible.

(c) A, B, and AB must all be invertible.

(d) A, B, AB, and A^T must all be invertible.

(e) A, B, AB, A^T, and $A + B$ must all be invertible.

12. $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^2$. Find the vector $\mathbf{x}$ in $\mathbb{R}^2$ that has $\mathcal{B}$-coordinates $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$.

(a) $\mathbf{x} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(b) $\mathbf{x} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

(c) $\mathbf{x} = \begin{bmatrix} 7 \\ -1 \end{bmatrix}$

(d) $\mathbf{x} = \begin{bmatrix} -7 \\ 1 \end{bmatrix}$

(e) None of the above.

13. Let the linear transformation $T(\mathbf{x}) = A\mathbf{x}$ be defined as $T(x_1, x_2, x_3) = (x_1 - 7x_2 + 4x_3, x_2 - 6x_3)$. Determine if T is one-to-one.

(a) T is one-to-one because the column vectors of A are not scalar multiples of each other.

(b) T is one-to-one because the column vectors of A span all of $\mathbb{R}^2$.

(c) T is one-to-one because $T(\mathbf{x}) = 0$ has only the trivial solution.

(d) T is not one-to-one because the column vectors of A are linearly independent.

(e) T is not one-to-one because the column vectors of A are linearly dependent.

14. Let $A = \begin{bmatrix} 4 & 2 & -3 \\ 3 & 4 & 1 \\ 4 & 1 & 5 \end{bmatrix}$. Then $\lambda = 3$ is an eigenvalue corresponding to the eigenvector $\mathbf{v} = \begin{bmatrix} 1 \\ -2 \\ a \end{bmatrix}$. Find the value of a.

(a) $a = 1$

(b) $a = -1$

(c) $a = 0$

(d) $a = 2$

(e) $a = -2$

15. Let $R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$. If R is the reduced row echelon form of the coefficient matrix for the system $A\mathbf{x} = \mathbf{0}$, what are the solutions to that system?

(a) $x_1 = 1$, $x_2 = 1$, and $x_3 = 2$

(b) $x_1 = 1$, $x_2 = 1$, $x_3 = 2$, and $x_4 = 0$

(c) $x_1 = t$, $x_2 = t$, $x_3 = 2t$, and $x_4 = t$

(d) $x_1 = -t$, $x_2 = -t$, $x_3 = -2t$, and $x_4 = t$

(e) There are no solutions to the system.

16. Suppose A is an $m \times n$ matrix. Which of the following statements is **false**?

(a) Col A is a subspace of $\mathbb{R}^n$.

(b) Nul A is a subspace of $\mathbb{R}^n$.

(c) If the equation $A\mathbf{x} = \mathbf{b}$ has a solution, then $\mathbf{b}$ must be in Col A.

(d) If Nul $A = \{\mathbf{0}\}$, then the columns of A must be linearly independent.

(e) If Col $A = \mathbb{R}^m$, then A must have a pivot position in every row.