Math 1226 **Common Final Exam** **Spring 2015**

Form A

Instructions: Fill in A, B or C in the Test Version section. Then enter your NAME, ID Number, CRN (under Class ID) and write A, B, or C (under Test ID) on the opscan sheet. Darken the appropriate circles below your ID number and Class ID (CRN). **Use a number 2 pencil**. Machine grading may ignore faintly marked circles.

Mark your answers to the test questions in rows 1 – 14 of the op scan sheet. Your score on this test will be the number of correct answers. You have one hour to complete this portion of the exam. Turn in the op scan sheet with your answers, this exam and all scrap paper at the end of this part of the final exam.

Exam Policies: You may not use a book, notes, formula sheet, or a calculator or computer. Giving or receiving unauthorized aid is an Honor Code Violation.

Signature: ______________________________

Name (printed): ______________________________

Student ID #: ______________________________

1. Evaluate $\int \theta \sin(2\theta) d\theta$.

(A) $-\frac{1}{2}\theta\cos(2\theta) + \frac{1}{4}\sin(2\theta) + C$

(B) $-\frac{1}{2}\theta\cos(2\theta) + \frac{1}{2}\sin(2\theta) + C$

(C) $-\frac{1}{2}\theta^2\cos^2(2\theta) + C$

(D) $-\frac{1}{2}\theta^2\sin(2\theta) - \theta^2\cos(2\theta) + C$

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2. What integral is obtained after applying an appropriate trigonometric substitution to $\int \frac{6}{9x^2+1}\, dx$?

(A) $\int \frac{6}{\sec^2(\theta)}\, d\theta$

(B) $\int \frac{\tan(\theta)}{\sec(\theta)}\, d\theta$

(C) $\int \frac{\cos(\theta)}{\sin^2(\theta)+1}\, d\theta$

(D) $\int 2\; d\theta$

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3. At what point does the parametric curve $x = t^2 - t + 1$, $y = 6t$ have a vertical tangent?

(A) $(1, 0)$

(B) $(\frac{1}{2}, 3)$

(C) $(\frac{3}{4}, 3)$

(D) The curve has no vertical tangent.

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4. Evaluate $\lim_{x\to\infty} (5 + e^{2x})^{\frac{1}{x}}$.

(A) 1

(B) 2

(C) e

(D) e^2

5. Let A and B denote the infinite series $A = \sum_{n=1}^{\infty} \frac{1}{(3n)!}$ and $B = \sum_{n=1}^{\infty} \frac{5(-1)^n}{2+3^n}$. Which of the following must be true?

(A) Both series converge.

(B) Both series diverge.

(C) A converges and B diverges.

(D) A diverges and B converges.

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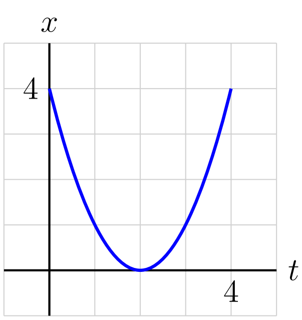
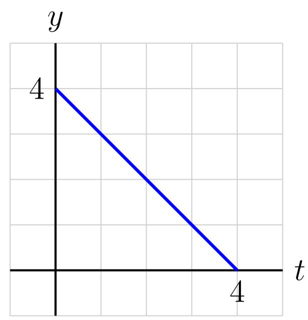
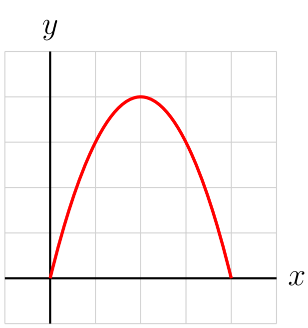
6. The graphs of the equations $x = f(t)$ and $y = g(t)$ are shown here over the interval $0 \leq t \leq 4$:

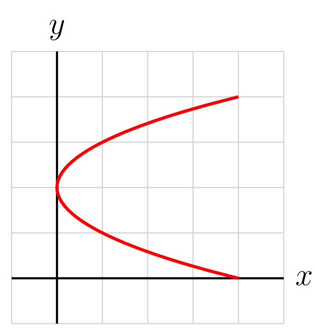




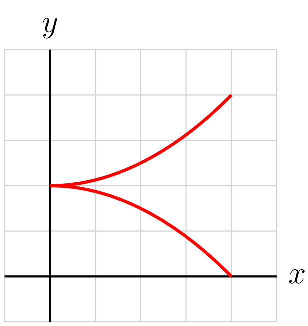
Which of the graphs below represents the graph of the parametric curve $x = f(t), y = g(t)$ in the xy-plane?



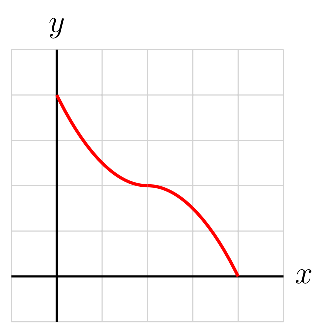
(A)



(C)



(B)



(D)

7. A manufacturer wants to know the volume of a bowl-shaped solid obtained by revolving around the y-axis the region bounded by the curves

$$x = 0, \qquad y = x^2 + 1, \qquad y = 2x^2$$

Which of the choices below correctly computes the volume?

(A) $2\pi \int_0^1 x(1 - x^2)\, dx$

(B) $\pi \int_0^2 (y/2) - (y - 1)\, dy$

(C) $2\pi \int_0^2 y(\sqrt{y/2} - \sqrt{y-1})\, dy$

(D) $\pi \int_0^1 (1 + x^2)^2 - (2x^2)^2\, dx$

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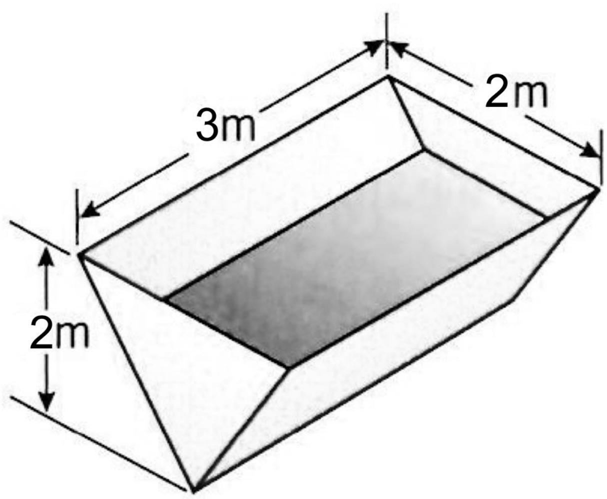
8. A reservoir completely filled with water is shaped as shown below. How much work is done by pumping all of the water over the top edge of the reservoir? (The density of water is 1000 kg/m^3.)



(A) $9.8 \cdot 2000$ J

(B) $9.8 \cdot 4000$ J

(C) $9.8 \cdot 5000$ J

(D) $9.8 \cdot 8000$ J

9. Let the region $\mathcal{R}$ be bounded by $x = 3y + 5$ and $x = y^2 + 1$. <u>SET UP ONLY</u> the $x-$coordinate of the centroid of the region with constant density ρ.

(A) $\bar{x} = \rho \int_{-1}^{4} y[(3y+5) - (y^2+1)]\, dy$

(B) $\bar{x} = \dfrac{\int_{-1}^{4} y[(3y+5) - (y^2+1)]\, dy}{\int_{-1}^{4} [(3y+5) - (y^2+1)]\, dy}$

(C) $\bar{x} = \dfrac{1}{2}\rho \int_{-1}^{4} [(3y+5)^2 - (y^2+1)^2]\, dy$

(D) $\bar{x} = \dfrac{\frac{1}{2}\int_{-1}^{4} [(3y+5)^2 - (y^2+1)^2]\, dy}{\int_{-1}^{4} [(3y+5) - (y^2+1)]\, dy}$

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10. A series $\sum_{n=1}^{\infty} a_n$ has k-th partial sum $\left(\dfrac{3k+2}{k}\right)^2$. Which of the following must be true?

(A) $\lim_{n\to\infty} a_n = 3$

(B) $\sum_{n=1}^{\infty} a_n = 9$

(C) The sequence $\{a_n\}$ diverges

(D) $\sum_{n=1}^{\infty} a_n = 3$

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11. Consider the function

$$f(x) = \begin{cases} 0, & x < 0 \\ \dfrac{c}{4+x^2}, & x \geq 0 \end{cases}$$

For what value of c will $f(x)$ be a probability density function?

(A) $\dfrac{\pi}{4}$ (B) $\dfrac{\pi}{2}$ (C) $\dfrac{2}{\pi}$ (D) $\dfrac{4}{\pi}$

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12. Find the Maclaurin series for $f(x) = xe^{-x^2}$.

(A) $\sum_{n=0}^{\infty} \dfrac{(-1)^n x^{2n+1}}{n!}$

(B) $\sum_{n=0}^{\infty} \dfrac{(-x^2)^{n+1}}{n!}$

(C) $\sum_{n=0}^{\infty} \dfrac{(-1)^n x^{2n+1}}{(n)!(2n+1)}$

(D) $\sum_{n=0}^{\infty} \dfrac{(-1)^n (x^2)^{n+1}}{(2n)!}$

13. Find the <u>radius of convergence</u> for the power series

$$\sum_{n=1}^{\infty} \frac{(x+4)^n}{(-2)^n \sqrt{n+1}}$$

(A) 1 (B) 2 (C) 4 (D) ∞

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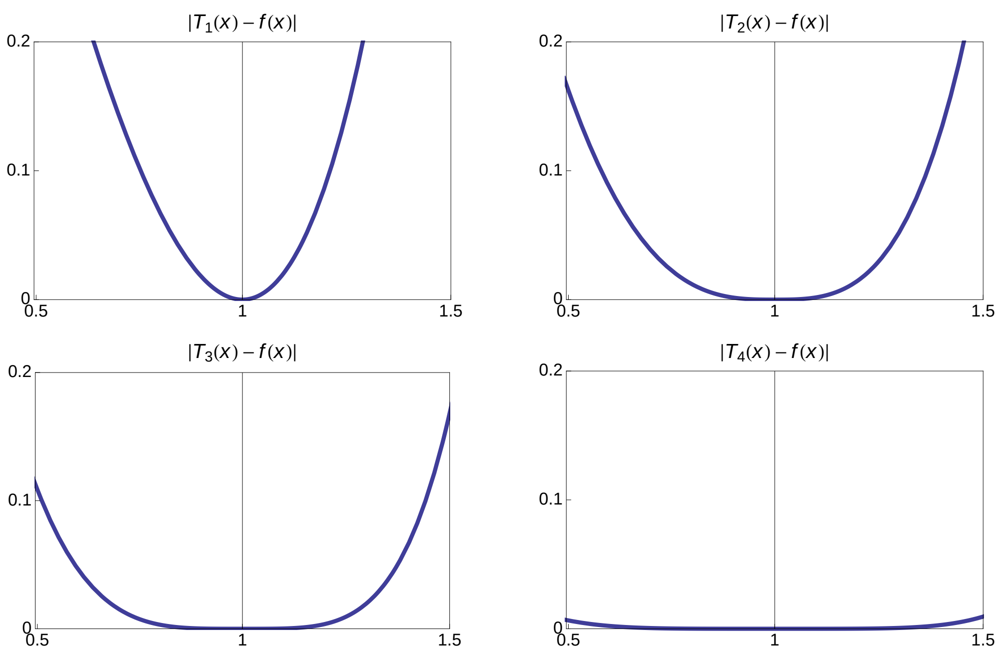
14. Suppose that $f(x)$ is a function with degree-four Taylor polynomial

$$T_4(x) = 4 + 2(x-1) - (x-1)^2 + 2(x-1)^3 + (x-1)^4.$$

Let T_k be the Taylor polynomial of <u>smallest</u> degree that approximates $f(x)$ so that

$$|T_k(x) - f(x)| < 0.2 \quad \text{for} \quad |x-1| < 0.5.$$

Use the graphs to determine k and decide whether each of statements I) and II) below is true or false.



$$I : T_k(0) = -1 \qquad II : \left| \int_1^{1.5} T_k(x)\,dx - \int_1^{1.5} f(x)\,dx \right| \leq 0.1$$

(A) I is true and II is false.

(B) I is false and II is true.

(C) I is true and II is true.

(D) I is false and II is false.