Math 2214, Spring 2014, Form A

1. Which of the following is a linear equation?

   (a) $(t^2y'' + y')/y = t^3$.

   (b) $y''' - y'' = ye^{y+t}$.

   (c) $y' = t/y$.

   (d) $y'' + y' = y^2$.

2. If $x' = x^2/t$, and $x(1) = 1$, then $x(2)$ is

   (a) $1/(1 - \ln 2)$.

   (b) $1/(1 + \ln 2)$.

   (c) $\sqrt{6}$.

   (d) $\sqrt{2}$.

3. A lake contains 10^8 gallons of water. The stream passing through the lake transports a constant 10^6 gallons per day. A plant situated just upstream of the lake starts releasing 100 grams per day of a toxic chemical. Let Q denote the amount of chemical in the lake measured in grams, and t the time in days measured from the time when the pollution started. Assume that the lake is well mixed. Then Q satisfies the following equation

   (a) $Q' = 10^6 - Q/10^8$, $Q(0) = 100$.

   (b) $Q' = 100 - Q/100$, $Q(0) = 10^8$.

   (c) $Q' = 100 - Q/100$, $Q(0) = 0$.

   (d) $Q' = -Q/100$, $Q(0) = 100$.

4. For the system

$$x' = -x + 5y,$$
$$y' = -7x + y,$$

the origin is a

(a) focus.

(b) saddle.

(c) center.

(d) node.

5. The general solution of the system $y' = Ay$, where

$$A = \begin{pmatrix} 0 & 4 \\ 1 & 0 \end{pmatrix},$$

is

(a) $c_1 e^{-2t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

(b) $c_1 e^{-2t} \begin{pmatrix} -2 \\ 1 \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

(c) $c_1 \cos(2t) \begin{pmatrix} 2 \\ 1 \end{pmatrix} + c_2 \sin(2t) \begin{pmatrix} 1 \\ -2 \end{pmatrix}$.

(d) $c_1 t e^{2t} \begin{pmatrix} 2 \\ 1 \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

6. A particular solution of the equation $y'' + y = e^t/t$ should have the form

(a) Ae^t/t.

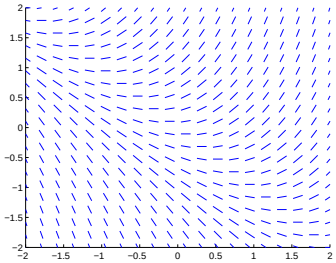
(b) Ae^t/t^2.

(c) $Ae^t + Be^t/t$.

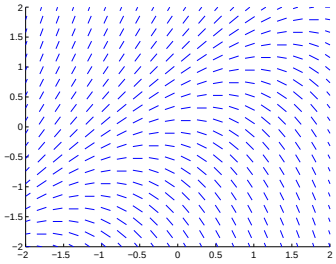
(d) $u(t) \sin t + v(t) \cos t$.

7. Which of the following is a direction field for the equation $y' = y + t^2$?

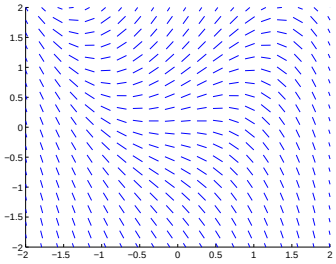
(a)

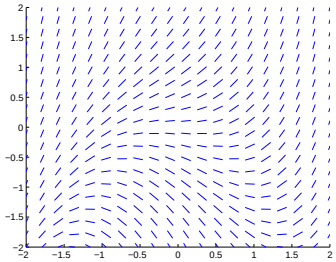


(b)



(c)



(d) 

8. Which of the following is not a particular solution of the equation $y'''' - y = e^t$?

   (a) $te^t/4 + 5e^t$.

   (b) $te^t/4 + e^{-t}$.

   (c) $te^t/4 + t^2e^t + 7e^t$.

   (d) $te^t/4 + 6e^t + 2\sin t$.

9. You solve the initial value problem $y' = y^2 + t$, $y(0) = 1$ using the Euler method with $h = 0.2$. Then the approximation you find for $y(0.4)$ is

   (a) 1.24.

   (b) 1.6275.

   (c) 1.528.

   (d) 1.2.

10. The matrix

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

    has how many linearly independent eigenvectors for the eigenvalue 1?

    (a) One.

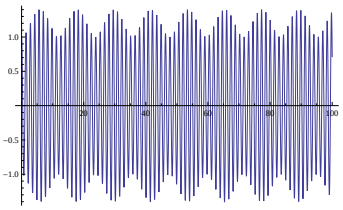
    (b) Three.

    (c) Four.

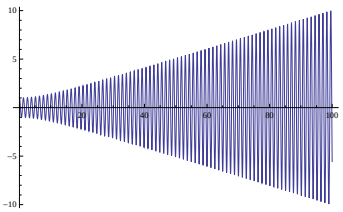
    (d) Two.

11. Which of the following plots shows the solution of the problem $y'' + 25y = \cos(5t)$, $y(0) = 1$, $y'(0) = 0$?

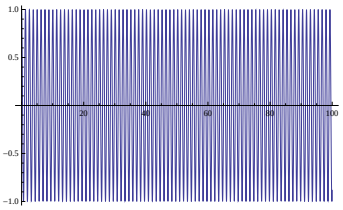
(a)

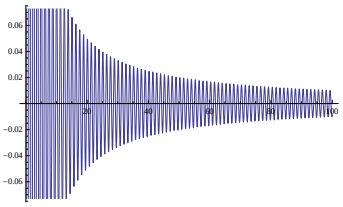


(b)



(c)



(d) 

12. A nonlinear system is given by

$$x_1' = x_1^2 x_2 - x_1.$$

$$x_2' = x_2 x_1 - x_1^2.$$

The linearization at the equilibrium point $(1, 1)$ is the system

(a)

$$y_1' = y_2,$$
$$y_2' = y_2.$$

(b)

$$y_1' = -y_1 + y_2,$$
$$y_2' = -y_1 + y_2.$$

(c)

$$y_1' = y_1 + y_2,$$
$$y_2' = y_2.$$

(d)

$$y_1' = y_1 + y_2,$$
$$y_2' = -y_1 + y_2.$$