Math 2114 **Common Final Exam** **Fall 2017**

Test Version A

Instructions: Fill in **A** in the Test Version section. Then enter your NAME, ID Number, CRN (under Class ID) and write **A** (under Test ID) on the op-scan sheet. Darken the appropriate circles below your ID number and Class ID (CRN). **Use a number 2 pencil**. Machine grading may ignore faintly marked circles.

Mark your answers to the test questions in rows 1–14 of the op-scan sheet. Your score on this test will be the number of correct answers. You have one hour to complete this portion of the exam. Turn in the op-scan sheet with your answers, this exam, and all scrap paper at the end of this part of the final exam.

Exam Policies: You may not use a book, notes, formula sheet, or a calculator or computer. Giving or receiving unauthorized aid is an Honor Code Violation.

Signature: ________________________________

Name (printed): ____________________________

Student ID #: _____________________________

1. The rank of the matrix $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 3 & 3 \end{bmatrix}$ is

(A) 0. (B) 1. (C) 2. (D) 3.

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2. If $A\mathbf{x} = \begin{bmatrix} 11 \\ 6 \end{bmatrix}$, and $A^{-1} = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$, then $\mathbf{x} =$

(A) $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$. (B) $\begin{bmatrix} -3 \\ -1 \end{bmatrix}$. (C) $\begin{bmatrix} 52 \\ 29 \end{bmatrix}$. (D) $\begin{bmatrix} 77 \\ 24 \end{bmatrix}$.

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3. Which of the following is a steady state vector for the Markov chain with transition matrix P?

$$P = \begin{bmatrix} 1/5 & 4/5 & 2/5 \\ 1/5 & 0 & 1/5 \\ 3/5 & 1/5 & 2/5 \end{bmatrix}$$

(A) $\begin{bmatrix} 7 \\ 3 \\ 8 \end{bmatrix}$ (B) $\begin{bmatrix} 4 \\ 0 \\ -4 \end{bmatrix}$ (C) $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ (D) $\begin{bmatrix} 1 \\ -5 \\ 3 \end{bmatrix}$

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4. Let V be a set of nonzero vectors in $\mathbb{R}^n$ and $S = \text{span}(V)$. Which of the following is **not** possible?

(A) V is a linearly independent set.

(B) S is a linearly independent set.

(C) Every vector in $\mathbb{R}^n$ is a linear combination of the vectors in V.

(D) Every vector in $\mathbb{R}^n$ is a linear combination of the vectors in S.

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5. Suppose $p(\lambda) = \lambda(\lambda-1)(\lambda+2)(\lambda+1)$ is the characteristic polynomial of a matrix A. Consider the statements:

   I: A must be a 4×4 matrix.

   II: A can be diagonalized.

   III: A is invertible.

   Which of the statements I, II, and III are false?

   (A) Only I is false.

   (B) Only II is false.

   (C) Only III is false.

   (D) All of I, II, and III are true.

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6. Determine whether or not V spans $\mathbb{R}^2$ and choose a correct justification:

$$V = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}, \text{ where } \mathbf{v}_1 = \begin{bmatrix} 3 \\ -1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 2 \\ 7 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 6 \\ 21 \end{bmatrix}$$

   (A) V spans $\mathbb{R}^2$ because its echelon form contains a leading entry in every column.

   (B) V spans $\mathbb{R}^2$ because $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3 = \mathbf{b}$ is consistent for all $\mathbf{b} \in \mathbb{R}^2$.

   (C) V does not span $\mathbb{R}^2$, because $\mathbf{v}_3$ is an scalar multiple of $\mathbf{v}_2$.

   (D) V does not span $\mathbb{R}^2$, because a set of three vectors can only span $\mathbb{R}^3$.

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7. The vectors $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$, and $\mathbf{v}_3 = \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$ form an orthogonal basis for $\mathbb{R}^3$. If $\mathbf{w} = \begin{bmatrix} 4 \\ -2 \\ 4 \end{bmatrix}$, then the coordinate vector, $[\mathbf{w}]_{\mathcal{B}}$, of $\mathbf{w}$ with respect to the basis $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is

   (A) $\begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$.

   (B) $\begin{bmatrix} 6 \\ -6 \\ 6 \end{bmatrix}$.

   (C) $\begin{bmatrix} 6/\sqrt{3} \\ -6/\sqrt{2} \\ 6/\sqrt{6} \end{bmatrix}$.

   (D) none of the above.

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8. Let $n \geq 2$. How many of the following statements are equivalent to the $n \times n$ matrix A being invertible?

- $\operatorname{rank}(A) \neq 0$.
- The equation $A\mathbf{x} = \mathbf{0}$ has only one solution
- $A = A^T$.
- The reduced row echelon form of A is I_n.
- The rows of A form a basis for $\mathbb{R}^n$.

(A) One (B) Two (C) Three (D) Four

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9. The matrix $A = \begin{bmatrix} 4 & -7 & 2 \\ 3 & 1 & 0 \\ -1 & c & -2 \end{bmatrix}$ is **not** invertible. Determine the value of c.

(A) -6 (B) 6 (C) 8 (D) 10

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10. If the system $x_1 - x_3 = 5$, $x_2 + x_3 = 3$, $4x_1 + 3x_2 = 7$ is written in matrix form $A\mathbf{x} = \mathbf{b}$, then the coefficient matrix A is

(A) $\begin{bmatrix} 1 & -1 & 5 \\ 1 & 1 & 3 \\ 4 & 3 & 7 \end{bmatrix}$.

(C) $\begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 4 & 3 \end{bmatrix}$.

(B) $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 4 & 3 & 0 \end{bmatrix}$.

(D) $\begin{bmatrix} 1 & 0 & -1 & 5 \\ 0 & 1 & 1 & 3 \\ 4 & 3 & 0 & 7 \end{bmatrix}$.

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11. Let $\mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$. The projection of $\mathbf{v}$ onto $\mathbf{u}$ is

(A) $\begin{bmatrix} 1/5 \\ 0 \\ -3/5 \end{bmatrix}$. (B) $\begin{bmatrix} 2/5 \\ 0 \\ -6/5 \end{bmatrix}$. (C) $\begin{bmatrix} 2/5 \\ 1/5 \\ 0 \end{bmatrix}$. (D) $\begin{bmatrix} 4/5 \\ 2/5 \\ 0 \end{bmatrix}$.

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12. Let $n \geq 2$. Which of the following is true for all $n \times n$ matrices A, B, and C, and $n \times n$ zero matrix O?

(A) $AB = BA$.

(B) If $AB = AC$ then $B = C$.

(C) If $AB = O$, then $A = O$ or $B = O$.

(D) None of the above.

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13. Suppose A is an $m \times n$ matrix, $\mathbf{x}$ and $\mathbf{y}$ are in $\mathbb{R}^n$, and $A\mathbf{x} = A\mathbf{y}$. Then which of the following statements must be true?

(A) A is invertible.

(B) $\mathbf{x} = \mathbf{y}$.

(C) $\mathbf{x}$ and $\mathbf{y}$ are in null(A).

(D) $\mathbf{y} - \mathbf{x}$ is in null(A).

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14. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. Let $\mathbf{u}$ and $\mathbf{v}$ be two distinct, nonzero vectors in $\mathbb{R}^n$. Which of the following is sufficient to concluded that T is **not** one-to-one?

(A) $T(3\mathbf{v}) = T(\mathbf{u})$

(B) $T(3\mathbf{v}) = 2T(\mathbf{u})$

(C) $T(\mathbf{v}) = 2T(\mathbf{v}) + T(\mathbf{u})$

(D) $T(3\mathbf{v}) = 2T(\mathbf{v}) + T(\mathbf{u})$
