# Form A

**Instructions**: Fill in A, B or C in the Test Version section. Then enter your NAME, ID Number, CRN (under Class ID) and write A, B, or C (under Test ID) on the op-scan sheet. Darken the appropriate circles below your ID number and Class ID (CRN). **Use a number 2 pencil**. Machine grading may ignore faintly marked circles.

Mark your answers to the test questions in rows 1–15 of the op-scan sheet. Your score on this test will be the number of correct answers. You have one hour to complete this portion of the exam. Turn in the op-scan sheet with your answers, this exam and all scrap paper at the end of this part of the final exam.

**Exam Policies**: You may not use a book, notes, formula sheet, or a calculator or computer. Giving or receiving unauthorized aid is an Honor Code Violation.

Signature: ______________________________

Name (printed): ______________________________

Student ID #: ______________________________

1. The integral $\int \frac{1}{\sqrt{x^2-1}}\,dx$ can be rewritten as which of the following?

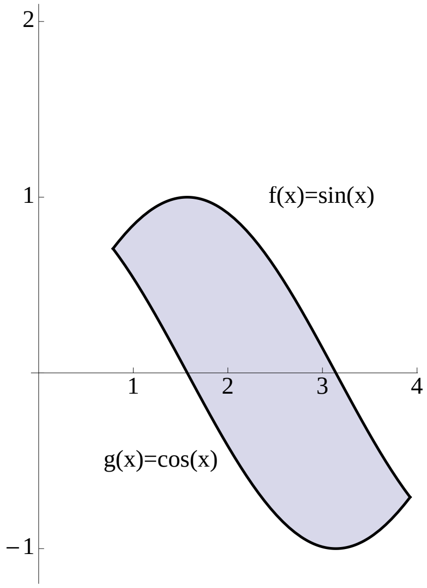
(A) $\int \sec(\theta)\,d\theta$ (B) $\int 1\,d\theta$ (C) $\int \cot(\theta)\,d\theta$ (D) $\int \frac{1}{\cos(\theta)\sin(\theta)}\,d\theta$

2. Evaluate $\int \frac{1}{x^3+x}\,dx$.

(A) $\ln|x| - \frac{1}{2}\ln|x^2+1| + C$ (B) $\frac{1}{2}\ln|x+1| + \frac{1}{2}\ln|x-1| - \ln|x| + C$

(C) $\ln|x| + C$ (D) $\ln|x| - \frac{1}{2x^2} + C$

3. Find the volume of the region graphed on the interval $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ below when it is rotated around $y = 2$.




(A) $\pi \int_{\pi/4}^{2\pi/4} \left((2-\cos(x))^2 - (2-\sin(x))^2\right)\, dx$

(B) $\pi \int_{\pi/4}^{3\pi/4} (x-2)(\sin(x) - \cos(x))\, dx$

(C) $\pi \int_{\pi/4}^{3\pi/4} \left(2^2 - (\sin(x) - \cos(x))^2\right)\, dx$

(D) $\pi \int_{-1}^{1} (y-2)(\arcsin(y) - \arccos(y))\, dy$

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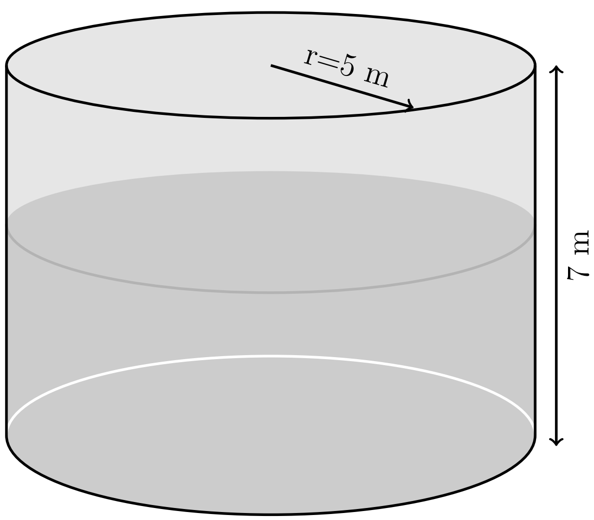
4. A cylindrical tank with height 7 meters and radius 5 meters is filled to a height of 4 meters with water, which has a density of 1000 kg/m$^3$. Set up an integral to find the amount of work required to pump the water out of a spout at the top of the cylindrical tank.




(A) $9800 \int_0^4 25\pi(7-y)dy$

(B) $9800 \int_0^4 \pi y^2(7-y)dy$

(C) $9800 \int_3^7 25\pi(4-y)dy$

(D) $9800 \int_4^7 25\pi(3-y)dy$

5. Set up an integral to find the moment about the $x$-axis, $M_x$, of the region bounded by $y = 2 - x^2$ and $y = x$ with density $\rho = 3$.

(A) $\displaystyle\int_{-2}^{1} \frac{3}{2}\Big((2-x^2)^2 - x^2\Big)\, dx$

(B) $\displaystyle\int_{-2}^{1} 3\Big((2-x^2) - x\Big)\, dx$

(C) $\displaystyle\int_{-2}^{1} 3\Big((2-x^2) - x\Big) \cdot x\, dx$

(D) $\displaystyle\int_{-2}^{1} \frac{3}{2}\Big((2-x^2) - x\Big)^2\, dx$

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6. Find the average value of $f(x) = x\sin(x)$ on $[0, \pi]$.

(A) $1$

(B) $\frac{\pi}{2}$

(C) $\pi$

(D) $\pi^2 - 4$

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7. Let $\displaystyle\sum_{n=1}^{\infty} a_n$ be a series such that $\displaystyle\lim_{n\to\infty} a_n = 0$ and $a_n > 0$. Let $s_k$ denote the $k$th partial sum, $s_k = \displaystyle\sum_{n=1}^{k} a_n$. Which of the following statements could be true about $s_k$?

(A) $\displaystyle\lim_{k\to\infty} s_k = 6.$

(B) $\displaystyle\lim_{k\to\infty} s_k = 0.$

(C) $s_k < 10$ for all $k$, but $\displaystyle\sum_{n=1}^{\infty} a_n$ diverges.

(D) $\displaystyle\lim_{k\to\infty} s_k = -3.$

8. Suppose a function $f(x)$ has the following Maclaurin series

$$f(x) = \sum_{n=0}^{\infty} \frac{(x-1)^n}{n!}.$$

Which of the following statements is true?

(A) $f'(x) = f(x)$ for all $x$.

(B) $f''(1) = \frac{1}{2!}$.

(C) $f'(1) = 0$

(D) $f(1) = 0$.

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9. Which of the following series converges conditionally?

(A) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\ln(n)+1}$

(B) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$

(C) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{2}$

(D) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{2n}}{n}$

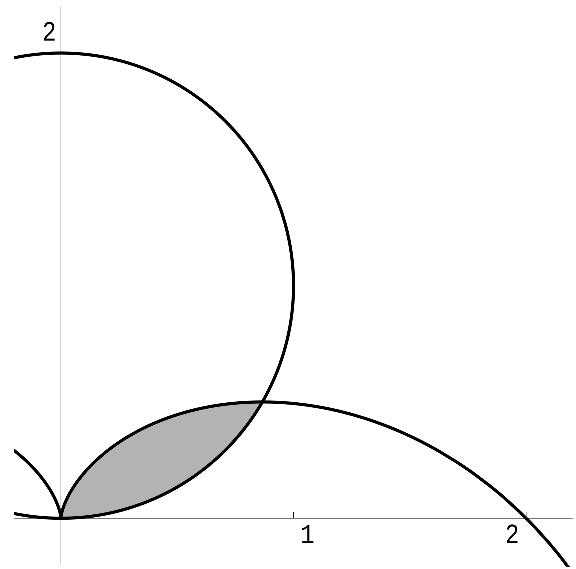
10. Let $r = 2 - 2\sin\theta$ and $r = 2\sin\theta$ be polar curves given in the graph below.
To find the area of the shaded region, we would use

(A) $\frac{1}{2}\int_0^{\pi/6} (2\sin\theta)^2\, d\theta + \frac{1}{2}\int_{\pi/6}^{\pi/2} (2 - 2\sin\theta)^2\, d\theta$

(B) $\frac{1}{2}\int_{\pi/6}^{\pi/2} (2 - 2\sin\theta)^2 - (2\sin\theta)^2\, d\theta$

(C) $\frac{1}{2}\int_0^{\pi/2} (2 - 2\sin\theta)^2 + (2\sin\theta)^2\, d\theta$

(D) $\frac{1}{2}\int_0^{\pi/6} 2\sin\theta\, d\theta + \frac{1}{2}\int_{\pi/6}^{\pi/2} 2 - 2\sin\theta\, d\theta$

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11. Use the trapezoidal rule with $n = 3$ to approximate $\ln(2)$.
(Recall that: $\ln(x) = \int_1^x \frac{1}{t} dt$).

(A) $\frac{7}{10}$ (B) $\frac{3}{2}$ (C) $\frac{7}{8}$ (D) $\frac{5}{21}$

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12. The **open** interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{(2x-4)^n}{3^n n^3}$$

is

(A) $x \in \left(\frac{1}{2}, \frac{7}{2}\right)$ (B) $x \in (-3, 3)$ (C) $x \in (-\infty, \infty)$ (D) $x \in \left(-\frac{3}{2}, \frac{3}{2}\right)$

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13. Which of the given series converges to the function $f(x) = \dfrac{2}{1 + 9x^2}$ on the appropriate interval of convergence.

(A) $\sum_{n=0}^{\infty} 2(-1)^n (3x)^{2n}$ (B) $\sum_{n=0}^{\infty} \frac{2x^{2n}}{3^n}$ (C) $\sum_{n=0}^{\infty} 2(3x)^{2n}$ (D) $\sum_{n=0}^{\infty} \frac{2(-1)^n}{(3x)^{2n}}$

14. Evaluate

$$\lim_{x\to 1^+} \frac{1}{x-1} - \frac{1}{\ln(x)}$$

(A) $-\frac{1}{2}$ (B) $0$ (C) $\infty$ (D) $e^{-1/2}$

15. Evaluate

$$\int_{-1}^{1} \frac{e^x}{e^x - 1}\, dx$$

(A) Diverges (B) $\ln\left|\frac{e-1}{1/e-1}\right|$ (C) $\ln\left|\frac{e-1}{1/e-1}\right| - 1$ (D) $1$

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16. Two students are told that the lifetime $T$ (in years) of an electric component is given by the exponential probability density function $f(t) = e^{-t}\quad t \geq 0$. They are asked to find time t=L where a typical component is 60% likely to exceed. The two students had the following approaches:

Student A: $0.6 = \int_L^\infty e^{-t} dt$

Student B: $0.6 = 1 - \int_0^L e^{-t} dt$

Which student has the correct setup to be able to find L?

(A) Both

(B) Student A

(C) Student B

(D) Neither Student