Math 1225 **Common Final Exam** **Fall 2017**

Form A

Instructions:

- Fill in A, B or C in the Test Version section.
- Enter your NAME, ID Number, CRN (under Class ID) and write A, B, or C (under Test ID) on the op-scan sheet.
- Darken the appropriate circles below your ID number and Class ID (CRN).
  Use a number 2 pencil. Machine grading may ignore faintly marked circles.
- Mark your answers to the test questions in rows 1–14 of the op-scan sheet. Your score on this test will be the number of correct answers.
- You have **one hour** to complete this portion of the exam. Turn in the op-scan sheet with your answers, this exam, and all scrap paper at the end of this part of the final exam.

Exam Policies: You may not use a book, notes, formula sheet, calculator, or a computer.

Name (printed): _____

Student ID #: _____

Honor Pledge: I have neither given nor received unauthorized assistance on this exam.

Signature: _____

1. Define the function $h(x) = \displaystyle\int_1^{\sqrt{x}} \cos(t^2)\,dt$. Find $h'(4)$.

(A) $\dfrac{1}{4}(\cos(4) - \cos(1))$

(B) $\dfrac{1}{4}(\sin(4) - \sin(1))$

(C) $\cos(4)$

(D) $\dfrac{\cos(4)}{4}$

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2. Suppose a function f satisfies

$$x^2 < f(x) < \tfrac{1}{2}x(x+1), \qquad \text{for } 0 < x < 1$$

$$f(x) = 2x + 1, \qquad \text{for } x \geq 1.$$

Which of the following statements **must be true**?

(A) $\lim\limits_{x \to 1^-} f(x)$ exists

(B) $f(0) = 0$

(C) $\lim\limits_{x \to \frac{1}{2}} f(x)$ does not exist

(D) f is continuous at $x = 1$

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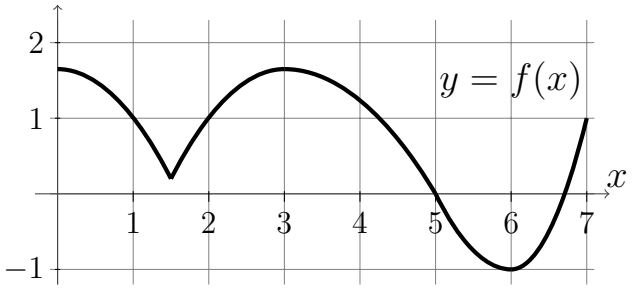
3. The graph of $y = f(x)$ is given below. If Newton's method is applied to the function f, starting with an initial guess of $x_1 = 1$, what happens to the iterates $x_2, x_3, x_4, \ldots$?



(A) $x_2 = 1.5$, $x_3 = 3$, and $x_4 = 6.75$.

(B) $x_2 = 6$ and x_3 cannot be calculated because $f'(x_2) = 0$.

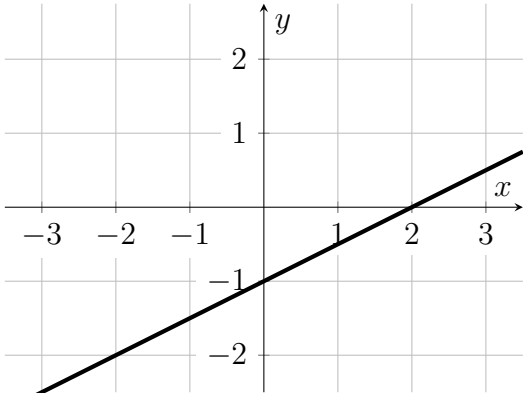
(C) The iterates $x_2, x_3, x_4, \ldots$ oscillate and do not approach any root.

(D) The iterates $x_2, x_3, x_4, \ldots$ will approach the root $x = 5$.

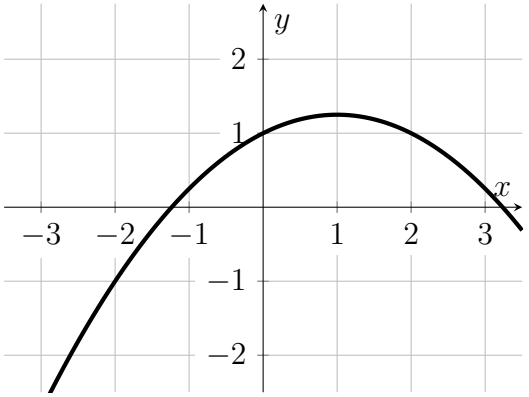
4. The linearization of a function f at $a = 2$ is given by $L(x) = \dfrac{1}{2}x$.

One of the choices below is the graph of f. Choose the correct graph of f.

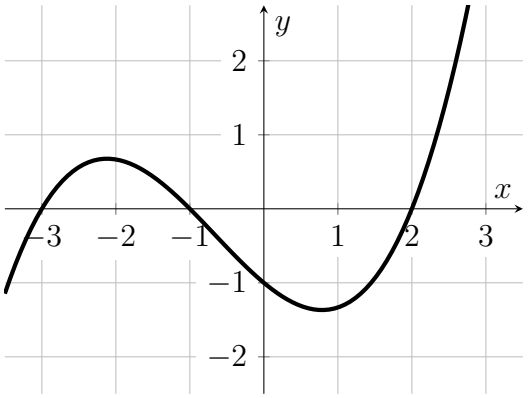
(A)



(C)



(B)



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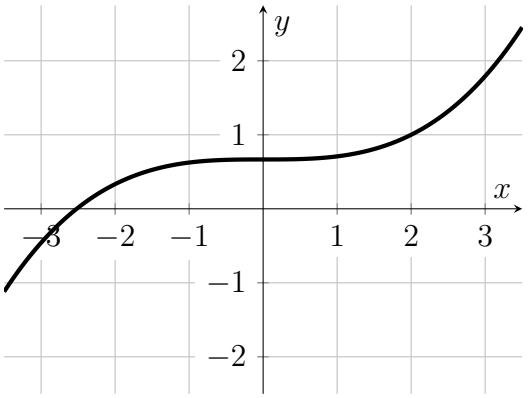

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5. Express the following integral as a limit of Riemman sums: $\displaystyle\int_2^5 (x^2 + 1)\, dx$

(A) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \left(\left(2 + \frac{i}{n} \right)^2 + 1 \right) \frac{1}{n}$

(B) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \left(\left(2 + \frac{3i}{n} \right)^2 + 1 \right) \frac{3}{n}$

(C) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \left(\left(\frac{i}{n} \right)^2 + 1 \right) \frac{3}{n}$

(D) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \left(3 + \frac{3i}{n} \right)^2 \frac{3}{n}$

6. After an appropriate u-substitution, which of the following integrals is equivalent to $\displaystyle\int_1^e \frac{(\ln(x))^2}{x}\,dx$?

(A) $\displaystyle\int_0^1 \frac{u^2}{e^u}\,du$ (B) $\displaystyle\int_1^e u^2\,du$ (C) $\displaystyle\int_0^1 u^2\,du$ (D) $\displaystyle\int_1^e u\ln(u)\,du$

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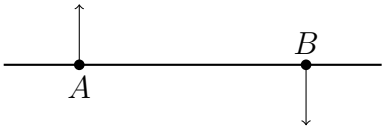
7. Two people, A and B, are standing 3 miles apart on a straight road. At the same time that person A walks north at a rate of 3 miles per hour (mph), person B walks south at a rate of 1 mph. At what rate is the distance between the two people changing 1 hour later?

(A) $\sqrt{10}$ mph (B) $\frac{16}{5}$ mph (C) $\frac{5}{2}$ mph (D) $\frac{16}{3}$ mph



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8. If the function f is differentiable on $[a, b]$, then which of the following statements **must be true**?

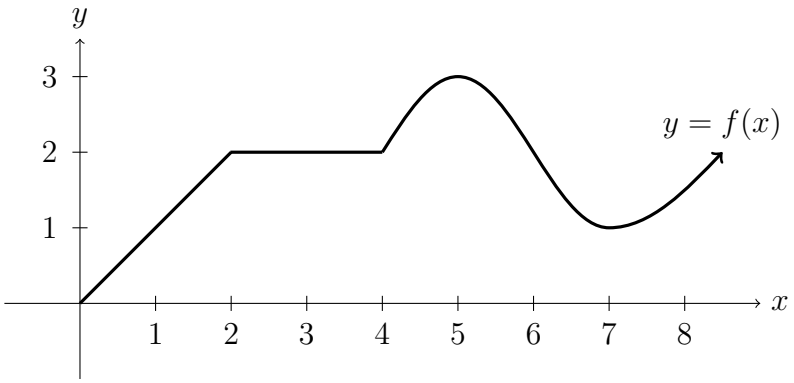
I. f has a maximum value on the open interval (a, b).

II. $f'(c) = \dfrac{f(b) - f(a)}{b - a}$ for some c such that $a < c < b$.

III. $f'(c) = 0$ for some c such that $a < c < b$.

(A) Only I (B) Only II (C) Only I and III (D) Only II and III

9. Consider the graph of $y = f(x)$ below, and suppose that $\int_2^8 f(x)\,dx = 11$.



The value of $\int_4^8 (2f(x) + 3)\,dx$ is

(A) 26 (B) 22 (C) 17 (D) 7

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10. Suppose

$$f'(a) = \lim_{h \to 0} \frac{\sqrt[3]{8 + 6h + h^2} - 2}{h}.$$

From the choices below, choose the correct function f and the corresponding value of a.

(A) $f(x) = \sqrt[3]{x^2 + 1}$, $a = 0$

(B) $f(x) = \sqrt[3]{x^2} + 2$, $a = 0$

(C) $f(x) = \sqrt[3]{x^2 - 1}$, $a = 3$

(D) $f(x) = 1 - \sqrt[3]{x^2}$, $a = 3$

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11. Consider the equation $y^2 + x^2 = 3xy - 4$. Find $\dfrac{dy}{dx}$ evaluated at the point $(2, 4)$.

(A) -2

(B) $-\dfrac{4}{5}$

(C) $\dfrac{1}{4}$

(D) 4

12. Let f be a function such that f'' is continuous on $(-\infty, +\infty)$, $f'(4) = 0$ and $f''(4) = -1$. Consider $g(x) = f(x) + (x-4)^2$. Which of the following can be determined about g at $x = 4$?

(A) g has a local minimum at $x = 4$.

(B) g has a local maximum at $x = 4$.

(C) g has an inflection point at $x = 4$.

(D) Nothing can be determined about g.

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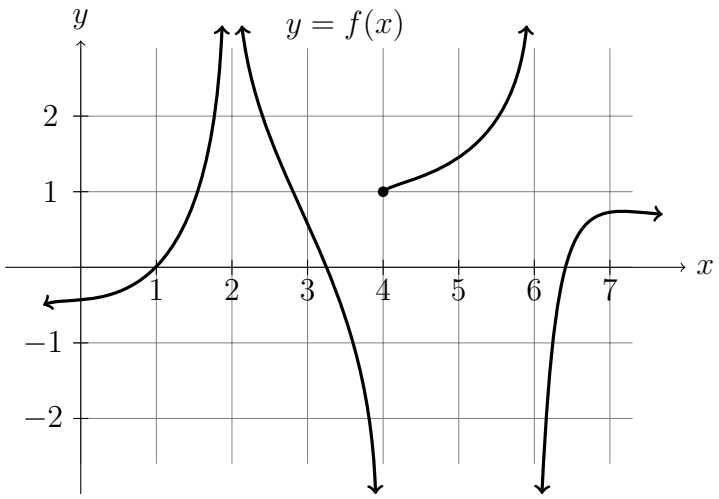
13. Consider the graph of $y = f(x)$ below. Which of the following statements is FALSE?



(A) The line $x = 4$ is a vertical asymptote.

(B) $\lim_{x \to 2} \dfrac{1}{f(x)} = 0$

(C) $\lim_{x \to 4^+} f(x) = 1$

(D) $\lim_{x \to 6} f(x) = -\infty$

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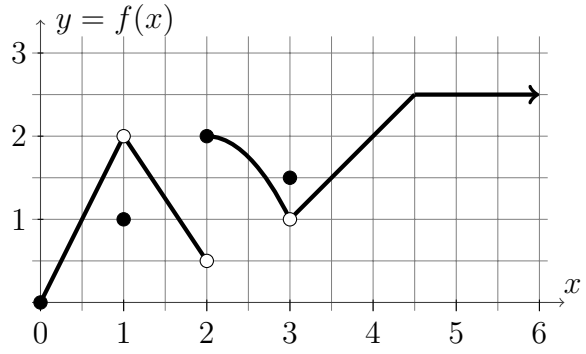
14. Consider the graph of $y = f(x)$ below.
Find the largest value of $\delta > 0$ such that if $0 < |x-4| < \delta$, then $|f(x) - 2| < 1$.



(A) $\delta = 1$

(B) $\delta = 2$

(C) $\delta = 3$

(D) no such δ exists because f is not continuous