Math 2204 Common Final Exam Fall 2016

Test Version: A

INSTRUCTIONS: Please enter your **NAME**, **ID Number**, **Test Version**, and your **CRN** on the opscan sheet. The CRN should be written in the field labeled **Class ID** and the test version (A, B, or C) under **Test ID**. Leave the Date, Instructor/Class, Test Name, and Time fields blank. Darken the appropriate circles below your ID number, below Class ID, and beside Test Version. **Use a number 2 pencil**. Machine grading may ignore faintly marked circles.

Mark your answers to the test questions in rows 1-14 of the opscan sheet. Your score on this test will be the number of correct answers. You have one hour to complete this portion of the exam. Turn in the opscan sheet with your answers and the question sheets at the end of this part of the final exam.

Exam Policies: You may not use a book, notes, formula sheet, calculator or a computer. Giving or receiving unauthorized aid is an Honor Code Violation.

Signature: ______________________________

Name (printed): ___________________________

Student ID: # ____________________________

[1] The point $(1, 0)$ is a critical point of the function $f(x, y) = 3x\mathrm{e}^y - x^3 - \mathrm{e}^{3y}$. Then

A) f has a local maximum at $(1, 0)$ B) f has a local minimum at $(1, 0)$

C) f has a saddle point at $(1, 0)$ D) the second derivative test is inconclusive at $(1, 0)$

[2] How many solutions (x, y, λ) does the following system of equations have?

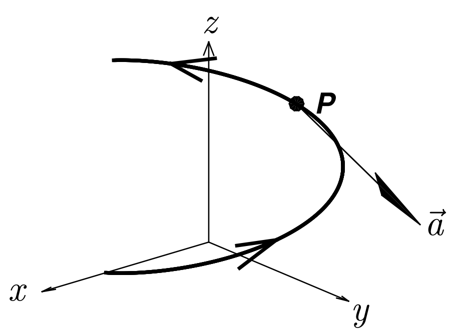
$$\begin{aligned} 2x &= \lambda x \\ y^2 &= \lambda \\ x + y^2 &= 4 \end{aligned}$$

A) 1 B) 2 C) 3 D) 4

[3] A differentiable function $f(x, y)$ has partial derivatives $f_x(1,1) = 2 - 2\sqrt{2}$ and $f_y(1,1) = -2$. Then the directional derivative at $(1,1)$ in the direction $\boldsymbol{i} + \boldsymbol{j}$ equals

A) -2 B) $-2\sqrt{2}$ C) $\langle \sqrt{2} - 2,\ -\sqrt{2} \rangle$ D) $\langle 2 - 2\sqrt{2},\ -2 \rangle$

[4] A particle has acceleration $\vec{a}$ at point P when moving along the smooth, oriented curve below. Which of the following statements is **true** about the unit tangent vector $\vec{T}$ at P and the binormal vector $\vec{B}$ at P?



A) $\vec{a} \cdot \vec{T} > 0$ and $\vec{a} \cdot \vec{B} = 0$.

B) $\vec{a} \cdot \vec{T} > 0$ and $\vec{a} \cdot \vec{B} < 0$.

C) $\vec{a} \cdot \vec{T} < 0$ and $\vec{a} \cdot \vec{B} = 0$.

D) $\vec{a} \cdot \vec{T} < 0$ and $\vec{a} \cdot \vec{B} < 0$.

[5] The direction of the line of intersection of the planes $2x + y - z = 3$ and $x + 2y + z = 3$ is

A) $\langle 1, -1, -2 \rangle$ B) $\langle 1, -1, 1 \rangle$ C) $\langle 3, 3, 0 \rangle$ D) $\langle 3, 3, 3 \rangle$

[6] The length of the curve $\boldsymbol{r}(t) = \langle\ 10 \sin t,\ -6 \cos t,\ 8 \cos t\ \rangle$ with $0 \le t \le \pi/2$ is

A) 10 B) $10\sqrt{2}$ C) 5π D) $5\pi\sqrt{2}$

[7] Compute $\displaystyle\int_0^2 \int_{\sqrt{x}}^{\sqrt{2}} y \sin(y^4)\ \mathrm{d}y\ \mathrm{d}x$.

A) $\dfrac{1}{4} - \dfrac{\cos(16)}{4}$ B) $\dfrac{1}{4} - \dfrac{\cos(4)}{4}$ C) $\dfrac{\sin(4)}{4}$ D) $\dfrac{\sin(16)}{4}$

[8] Let $f(x,y)$ be a function such that

- The limit of $f(x,y)$ as $x \to 0$ along the path $y = x$ is 0.
- The limit of $f(x,y)$ as $x \to 0$ along the path $y = x^2$ is 0.
- $f(0,0) = 1$.

Which of the following statements **must be true**?

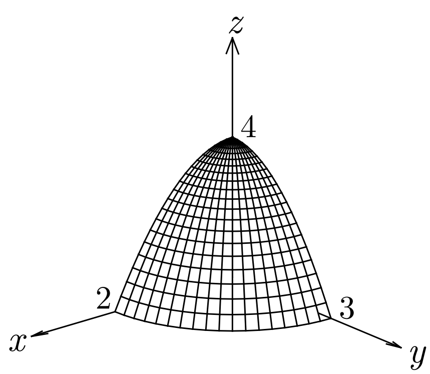
A) There is not enough information given to determine if $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ exists.

B) $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ does not exist.

C) $\lim\limits_{(x,y)\to(0,0)} f(x,y) = 0$.

D) $\lim\limits_{(x,y)\to(0,0)} f(x,y) = 1$.

[9] Determine the signs of f_x and f_y for the function $f(x,y)$ whose graph is shown below.



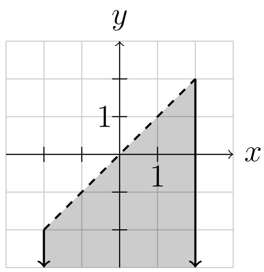
A) $f_x(1{,}1)$ is negative and $f_y(1,1)$ is positive.

B) $f_x(1{,}1)$ is positive and $f_y(1,1)$ is negative.

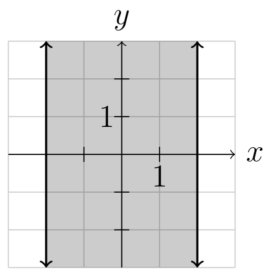
C) Both $f_x(1{,}1)$ and $f_y(1{,}1)$ are positive.

D) Both $f_x(1,1)$ and $f_y(1,1)$ are negative.

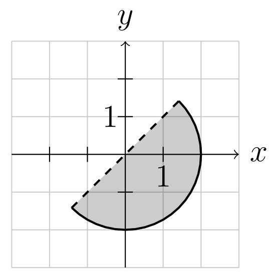
[10] Which of the following shows the domain of $f(x,y) = \ln(x-y)\sqrt{4-x^2}$?



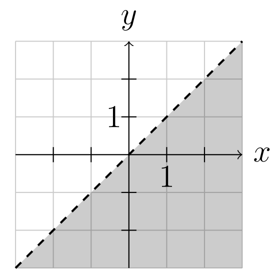
A)



B)



C)



D)

[11] Let E be the solid bounded by the plane $z = 6y$ and the paraboloid $z = x^2 + y^2$. Choose the iterated integral that correctly represents the volume of the solid E

A) $\displaystyle\int_0^6 \int_0^{\sqrt{6y-y^2}} \int_{x^2+y^2}^{6y} 1 \, dz \, dx \, dy$

B) $\displaystyle\int_0^6 \int_{-\sqrt{6y-y^2}}^{\sqrt{6y-y^2}} \int_{x^2+y^2}^{6y} 1 \, dz \, dx \, dy$

C) $\displaystyle\int_{-3}^3 \int_0^{\sqrt{9-y^2}} \int_{x^2+y^2}^{6y} 1 \, dz \, dx \, dy$

D) $\displaystyle\int_{-3}^3 \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} \int_{x^2+y^2}^{6y} 1 \, dz \, dx \, dy$

[12] The integral $\displaystyle\int_0^{\pi/2} \int_{3\pi/4}^{\pi} \int_0^{4\sqrt{2}} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$ converted to cylindrical coordinates is:

A) $\displaystyle\int_0^{\pi/2} \int_0^4 \int_{-r}^{-\sqrt{32-r^2}} r \, dz \, dr \, d\theta$

B) $\displaystyle\int_0^{\pi/2} \int_0^{4\sqrt{2}} \int_{-r}^{-\sqrt{32-r^2}} r \, dz \, dr \, d\theta$

C) $\displaystyle\int_0^{\pi/2} \int_0^{4\sqrt{2}} \int_{-\sqrt{32-r^2}}^{-r} r \, dz \, dr \, d\theta$

D) $\displaystyle\int_0^{\pi/2} \int_0^4 \int_{-\sqrt{32-r^2}}^{-r} r \, dz \, dr \, d\theta$

[13] A lamina lies in the region in the xy-plane where $y \geq 0$ and $x^2 + y^2 \leq 4$. The density of the lamina is given by $\rho(x, y) = \sqrt{x^2 + y^2}$. Which expression correctly represents the y-coordinate of the center of mass of the lamina?

A) $\displaystyle\frac{3}{8\pi} \int_0^{\pi} \int_0^2 r^3 \sin\theta \, dr \, d\theta$

B) $\displaystyle\frac{3}{8\pi} \int_0^{\pi} \int_0^2 r^2 \sin\theta \, dr \, d\theta$

C) $\displaystyle\frac{1}{2\pi} \int_0^{\pi} \int_0^2 r^3 \sin\theta \, dr \, d\theta$

D) $\displaystyle\frac{1}{2\pi} \int_0^{\pi} \int_0^2 r^2 \sin\theta \, dr \, d\theta$

[14] The line that is normal (perpendicular) to the surface $3x^2 - y^2 - 2z^2 = 3$ at the point $(3, 4, 2)$ intersects the yz-plane. What is the z-coordinate of this point of intersection?

A) -2 B) 0 C) 2 D) $\displaystyle\frac{10}{3}$