Form A

Instructions: Fill in A, B or C in the Test Version section. Then enter your NAME, ID Number, CRN (under Class ID) and write A, B, or C (under Test ID) on the op-scan sheet. Darken the appropriate circles below your ID number and Class ID (CRN). **Use a number 2 pencil**. Machine grading may ignore faintly marked circles.

Mark your answers to the test questions in rows 1–14 of the op-scan sheet. Your score on this test will be the number of correct answers. You have one hour to complete this portion of the exam. Turn in the op-scan sheet with your answers, this exam, and all scrap paper at the end of this part of the final exam.

Exam Policies: You may not use a book, notes, formula sheet, or a calculator or computer. Giving or receiving unauthorized aid is an Honor Code Violation.

Signature: ______________________________

Name (printed): ______________________________

Student ID #: ______________________________

1. A basis for the column space of

$$A = \begin{bmatrix} 4 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

is

(A) $\left\{ \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

(B) $\left\{ \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

(C) $\left\{ \begin{bmatrix} 4 \\ 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

(D) $\left\{ \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

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2. Consider the discrete dynamical system $\mathbf{x_{n+1}} = A\mathbf{x_n}$. Then $\lim_{x\to\infty} \mathbf{x_n} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ for which of the following?

(A) $A = \begin{bmatrix} 0.5 & .75 \\ 0 & -2 \end{bmatrix}$

(B) $A = \begin{bmatrix} 0.5 & 0 \\ .75 & -2 \end{bmatrix}$

(C) $A = \begin{bmatrix} 0.5 & .75 \\ 0 & -0.2 \end{bmatrix}$

(D) $A = \begin{bmatrix} 1.5 & 0 \\ .75 & -2 \end{bmatrix}$

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3. Let V be a subspace of $\mathbb{R}^n$ and $\mathbf{v_1}, \mathbf{v_2}, ..., \mathbf{v_m}$ a set of vectors in V. Suppose every vector in V may be expressed as a linear combination of $\mathbf{v_1}, \mathbf{v_2}, ..., \mathbf{v_m}$. Then:

(A) $\mathbf{v_1}, \mathbf{v_2}, ..., \mathbf{v_m}$ span $\mathbb{R}^n$.

(B) $\mathbf{v_1}, \mathbf{v_2}, ..., \mathbf{v_m}$ are linearly dependent.

(C) $\mathbf{v_1}, \mathbf{v_2}, ..., \mathbf{v_m}$ is a basis for V.

(D) $\mathbf{v_1}, \mathbf{v_2}, ..., \mathbf{v_m}$ span V.

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4. Suppose $\mathbf{v} = \begin{bmatrix} -1 \\ a \\ 8 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix}$. Determine the positive value of a such that the distance between $\mathbf{v}$ and $\mathbf{w}$ is 11.

(A) $a = 8$

(B) $a = 9$

(C) $a = 10$

(D) No such a is possible

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5. Suppose $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation such that

$$T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ -4 \end{bmatrix}, \text{ and } T\left(\begin{bmatrix} 3 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 3 \end{bmatrix}.$$

Then $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)$ is

(A) $\begin{bmatrix} -5 \\ 11 \end{bmatrix}$ (B) $\begin{bmatrix} 2 \\ -4 \end{bmatrix}$ (C) $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$ (D) $\begin{bmatrix} 3/2 \\ 2 \end{bmatrix}$

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6. Suppose $B = \begin{bmatrix} 1 & 0 & -3 & 0 & 0 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$. If B is the row-reduced echelon form of the matrix A, what are the solutions to the matrix equation $Ax = 0$?

(A) $\left\{ s\begin{bmatrix} 3 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} \middle| \, s, t \in \mathbb{R} \right\}$

(C) $\left\{ s\begin{bmatrix} -3 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \middle| \, s, t \in \mathbb{R} \right\}$

(B) $\left\{ s\begin{bmatrix} 3 \\ -2 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix} \middle| \, s \in \mathbb{R} \right\}$

(D) $\left\{ s\begin{bmatrix} 3 \\ -2 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} \middle| \, s \in \mathbb{R} \right\}$

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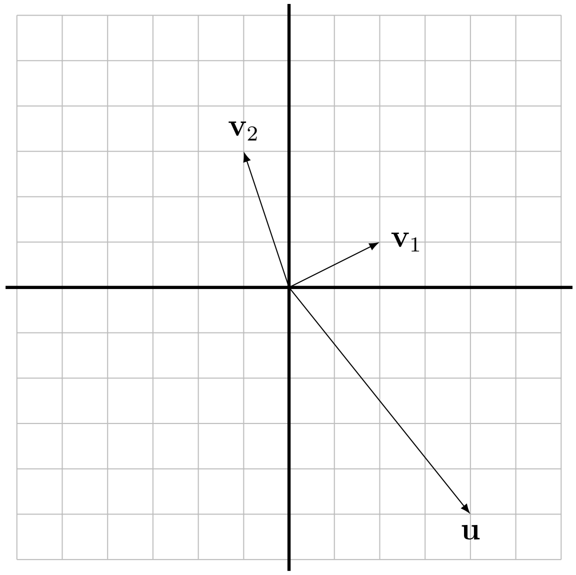
7. Vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{u}$ are shown below:



Let $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2\}$. Find the coordinates of $[\mathbf{u}]_{\mathcal{B}}$.

(A) $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ (B) $\begin{bmatrix} 3 \\ -12 \end{bmatrix}$ (C) $\begin{bmatrix} 7 \\ -14 \end{bmatrix}$ (D) $\begin{bmatrix} 13 \\ -11 \end{bmatrix}$

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8. Let $A = \begin{bmatrix} a & b & 0 & c \\ 0 & d & 0 & 0 \\ e & f & g & h \\ i & j & 0 & k \end{bmatrix}$, where a, b, c, d, e, f, g, h, i, j, and k are nonzero constants. Find $\det A$.

(A) $adgk$ (B) $adgk - cdgi$ (C) $dgk + dgi$ (D) 0

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9. Let $S = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \text{ in } \mathbb{R}^2 : xy \geq 0 \right\}$. Determine whether S is a subspace of $\mathbb{R}^2$.

(A) S is a subspace of $\mathbb{R}^2$.

(B) S is not a subspace of $\mathbb{R}^2$ because it does not contain the zero vector.

(C) S is not a subspace of $\mathbb{R}^2$ because it is not closed under vector addition.

(D) S is not a subspace of $\mathbb{R}^2$ because it is not closed under scalar multiplication.

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10. The linear transformation $T(\mathbf{x}) = A\mathbf{x}$ scales a vector $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ in $\mathbb{R}^2$ by 200% in the horizontal direction and 500% in the vertical direction, then rotates the vector 90° clockwise. Determine the standard matrix A of this transformation.

(A) $A = \begin{bmatrix} 2 & 1 \\ -1 & 5 \end{bmatrix}$ (B) $A = \begin{bmatrix} 0 & 5 \\ -2 & 0 \end{bmatrix}$ (C) $A = \begin{bmatrix} 0 & 2 \\ -5 & 0 \end{bmatrix}$ (D) $A = \begin{bmatrix} 2 & -5 \\ 5 & 2 \end{bmatrix}$

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11. Let A be an $m \times n$ matrix with $m < n$. Which of the following statements is always **false**?

(A) The rank of $A < m$.

(B) The nullity of $A = m$.

(C) The rank of $A = 0$.

(D) The nullity of $A = 0$.

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12. A is a 4×5 matrix which is row-equivalent to $\begin{bmatrix} 1 & 0 & 0 & 0 & -5 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 6 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$. Which of these statements about the linear transformation $T(\mathbf{x}) = A\mathbf{x}$ is **true**?

(A) T is neither one-to-one nor onto, because A is not square.

(B) T is one-to-one, because A has a pivot in every row.

(C) T is one-to-one, because A has a pivot in every column.

(D) T is onto, because A has a pivot in every row.

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13. Let $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$, and $\mathbf{a}_4$ be vectors in $\mathbb{R}^4$ and let $A = [\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3\ \mathbf{a}_4]$. Which of these statements, if true, would prove that A is **not** invertible?

(A) rank $[\mathbf{a}_1\ \mathbf{a}_2]$ = rank $[\mathbf{a}_3\ \mathbf{a}_4]$

(B) $\mathbf{a}_1 + \mathbf{a}_2 = \mathbf{a}_3 + \mathbf{a}_4$

(C) The matrix $B = [\mathbf{a}_4\ \mathbf{a}_3\ \mathbf{a}_2\ \mathbf{a}_1]$ is invertible.

(D) A^T has only one eigenvalue.

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14. Let $A = \begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$. Determine the eigenvalue associated with $\mathbf{v} = \begin{bmatrix} 1+i \\ i-1 \end{bmatrix}$, which is an eigenvector of A.

(A) $3 - 4i$ (B) $3 + 4i$ (C) $7 - i$ (D) $7 + i$
