



Math 2114
Common Final Exam
December 16, 2014
Form A

Instructions: Using a #2 pencil only, write your name and your instructor's name in the blanks provided. Write your student ID number and your CRN in the blanks marked "ID number" and "Class ID", and bubble in the appropriate spaces on the form.

Important: Mark your form letter **A** at the very top of your form next to the words "Test Version". Your form cannot be processed without this information.

You have one hour to complete this part of the final exam. Mark your answers to the test questions in rows 1–15 of the answer sheet. Your score on this part of the exam will be the number of correct answers. There is no penalty for guessing.

At the end of the hour, turn in both this test with the cover sheet intact, and the answer sheet.

Honor Policy: You may not use a book, notes, formula sheet, or any electronic device during this exam. Giving or receiving unauthorized aid is a violation of the undergraduate honor code.

Name: _____

Signature: _____

Student ID number: _____

1. Let

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$$

and let $H = \text{span } \mathcal{B}$. Let \mathbf{x} have coordinates x_1, x_2, x_3 in the standard basis. To find $[\mathbf{x}]_{\mathcal{B}}$ we multiply

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

on the left by which of the following matrices?

(a) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & -1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$

(e) $\begin{bmatrix} 0 & 0 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

2. Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be the linear transformation defined by

$$T(x_1, x_2) = (x_1 - 2x_2, x_1, 3x_1 + x_2).$$

Which of the following is the standard matrix for the linear transformation T ?

(a) $\begin{bmatrix} 1 & -2 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} -2 & 1 \\ 0 & 1 \\ 1 & 3 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 3 \\ 1 & 0 \\ -2 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 1 & 3 \\ -2 & 0 & 1 \end{bmatrix}$

(e) $\begin{bmatrix} -2 & 0 & 1 \\ 1 & 1 & 3 \end{bmatrix}$

3. Determine whether the set $\left\{ \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ 9 \end{bmatrix}, \begin{bmatrix} -5 \\ 1 \\ 13 \end{bmatrix} \right\}$ is a basis for \mathbb{R}^3 .

- (a) The set is a basis for \mathbb{R}^3 .
- (b) The set is not a basis for \mathbb{R}^3 as the set is linearly independent.
- (c) The set is not a basis for \mathbb{R}^3 as it is not closed under scalar multiplication.
- (d) The set is not a basis for \mathbb{R}^3 as the set spans all of \mathbb{R}^3 .
- (e) The set is not a basis for \mathbb{R}^3 as the set does not span all of \mathbb{R}^3 .

4. Suppose $A = [\mathbf{a}_1 \ \mathbf{a}_2 \ \cdots \ \mathbf{a}_n]$ is an $n \times n$ invertible matrix, and \mathbf{b} is a non-zero vector in \mathbb{R}^n . Which of the following statements is **false**?

- (a) \mathbf{b} is a linear combination of $\{\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n\}$.
- (b) The determinant of A is nonzero.
- (c) $\text{rank } A = n$.
- (d) If $A\mathbf{b} = \lambda\mathbf{b}$ for some constant λ , then $\lambda \neq 0$.
- (e) \mathbf{b} is a vector in $\text{Nul } A$.

5. A matrix A can be row reduced to the following echelon form

$$\begin{bmatrix} 1 & 1 & 3 & 2 & 1 \\ 0 & 1 & 2 & 1 & 3 \\ 0 & 0 & 0 & a & b \\ 0 & 0 & 0 & 0 & c \end{bmatrix}.$$

Depending on the values of the constants a, b and c the rank of A

- (a) must be 2.
 - (b) must be 3.
 - (c) must be 4.
 - (d) can be 2, 3 or 4.
 - (e) can be 2, 3, 4 or 5.
6. Suppose

$$\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = 1.$$

Find

$$\det \begin{bmatrix} 2a & 2b & 2c \\ g & h & i \\ d+g & e+h & f+i \end{bmatrix}.$$

- (a) $-\frac{1}{2}$
- (b) $\frac{1}{2}$
- (c) -2
- (d) 2
- (e) 1

7. Consider the matrix $A = \begin{bmatrix} 1 & 3 & 1 & 0 \\ 0 & 2 & 4 & -2 \end{bmatrix}$ whose reduced echelon form is $\begin{bmatrix} 1 & 0 & a & b \\ 0 & 1 & c & d \end{bmatrix}$.

Find $a + b + c + d$:

- (a) -7
- (b) -4
- (c) -3
- (d) -1
- (e) 0

8. Let A be a 3×4 matrix whose column space has dimension 3. Consider the following matrices.

I.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

II.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 2 & 0 & 1 \end{bmatrix}$$

III.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Which of above matrices could possibly be the reduced row echelon form of A ?

- (a) **I** only
- (b) **II** only
- (c) **III** only
- (d) **I** and **II** only
- (e) **II** and **III** only

9. Suppose that A is an $m \times n$ matrix and the equation $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution. Which of the following statements must be true for all \mathbf{b} in \mathbb{R}^m ?

- (a) $A\mathbf{x} = \mathbf{b}$ must have at least one solution.
- (b) $A\mathbf{x} = \mathbf{b}$ has no solution.
- (c) $A\mathbf{x} = \mathbf{b}$ has a unique solution.
- (d) $A\mathbf{x} = \mathbf{b}$ does not have a unique solution.
- (e) $A\mathbf{x} = \mathbf{b}$ has the trivial solution.

10. Suppose you wish to determine whether a set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly independent. You form the matrix $A = [\mathbf{v}_1 \ \mathbf{v}_2 \ \mathbf{v}_3 \ \mathbf{v}_4]$ and you calculate its reduced row echelon form,

$$R = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \text{ You now decide to write } \mathbf{v}_2 \text{ as a linear combination of } \mathbf{v}_1, \mathbf{v}_3, \text{ and } \mathbf{v}_4.$$

Which is a correct linear combination?

- (a) $\mathbf{v}_2 = 3\mathbf{v}_3 + \mathbf{v}_4$
 - (b) $\mathbf{v}_2 = -3\mathbf{v}_3 - \mathbf{v}_4$
 - (c) $\mathbf{v}_2 = \mathbf{v}_4 - 3\mathbf{v}_3$
 - (d) $\mathbf{v}_2 = -\mathbf{v}_1 + \mathbf{v}_4$
 - (e) \mathbf{v}_2 cannot be written as a linear combination of $\mathbf{v}_1, \mathbf{v}_3,$ and \mathbf{v}_4 .
11. You are given that

$$\mathbf{x}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \mathbf{x}_3 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, P = [\mathbf{x}_1 \ \mathbf{x}_2 \ \mathbf{x}_3], D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & -4 \end{bmatrix},$$

and $A = PDP^{-1}$. Then $A^2\mathbf{x}_2 =$

- (a) $\begin{bmatrix} 25 \\ 0 \\ 50 \end{bmatrix}$.
- (b) $\begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$.
- (c) $\begin{bmatrix} 5 \\ 0 \\ 20 \end{bmatrix}$.
- (d) $\begin{bmatrix} 25 \\ 0 \\ 100 \end{bmatrix}$.
- (e) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

12. Find the eigenvalues of the following matrix.

$$A = \begin{bmatrix} -4 & 6 \\ -1 & 1 \end{bmatrix}$$

- (a) $\{-2, -1\}$
- (b) $\{-4, 6\}$
- (c) $\{-4, 1\}$
- (d) $\{1, 2\}$
- (e) $\{-1, 2\}$

13. Given that A is a 5×7 matrix with rank 3, which of the following is a true statement?
- (a) Nul A is a 2-dimensional subspace of \mathbb{R}^7 .
 - (b) Nul A is a 3-dimensional subspace of \mathbb{R}^7 .
 - (c) Nul A is a 4-dimensional subspace of \mathbb{R}^7 .
 - (d) Nul A is a 2-dimensional subspace of \mathbb{R}^5 .
 - (e) Nul A is a 3-dimensional subspace of \mathbb{R}^5 .
14. Determine if the given statement is true or false, and choose the reasoning that correctly supports your choice.
- If A is a 3×4 matrix, then the transformation $\mathbf{x} \mapsto A\mathbf{x}$ is one-to-one.
- (a) True. The columns of A are linearly independent.
 - (b) False. The columns of A are linearly dependent.
 - (c) True. The columns of A span \mathbb{R}^3 .
 - (d) False. The columns of A do not span \mathbb{R}^3 .
 - (e) It cannot be determined unless A is known.
15. What is the range of T , where $T(\mathbf{x}) = A\mathbf{x}$, and $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -2 & 0 \end{bmatrix}$?
- (a) A plane in \mathbb{R}^3
 - (b) A line in \mathbb{R}^3
 - (c) A line in \mathbb{R}^2
 - (d) All of \mathbb{R}^2
 - (e) All of \mathbb{R}^3