Math 1226 **Common Final Exam** **Fall 2014**

Form A

Instructions: Please enter your NAME, ID Number, FORM DESIGNATION (A, B, or C), and your CRN on the opscan sheet. The CRN should be written in the box labeled COURSE. Do not include the course number. Darken the appropriate circles below your ID number and below the Form designation letter. **Use a number 2 pencil**. Machine grading may ignore faintly marked circles.

Mark your answers to the test questions in rows 1 – 13 of the op scan sheet. Your score on this test will be the number of correct answers. You have one hour to complete this portion of the exam. Turn in the op scan sheet with your answers, this exam and all scrap paper at the end of this part of the final exam.

Exam Policies: You may not use a book, notes, formula sheet, or a calculator or computer. Giving or receiving unauthorized aid is an Honor Code Violation.

Signature: _____

Name (printed): _____

Student ID #: _____

1. Let $f(x) = kx(1-x)$ on the interval $[0,1]$ and $f(x) = 0$ for all other input values. For which value of k is $f(x)$ a probability density function?

(A) 1 (B) 6 (C) 3 (D) 2

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2. The area bounded by the curves $y = x\sqrt{4-x^2}$ and $y = 0$ equals

(A) 5 (B) 6 (C) $\frac{17}{3}$ (D) $\frac{16}{3}$

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3. Determine the value of the improper integral, if convergent:

$$\int_{-\infty}^{-1} x^2 e^{x^3}\, dx$$

(A) $\frac{1}{e}$ (B) $\frac{1}{3e}$ (C) $\frac{e}{3}$ (D) The integral diverges

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4. Which substitution would simplify the integral $\int \frac{x^4 dx}{\sqrt{4+x^2}}$?

(A) $x = 4\tan(\theta)$
(B) $x = 2\sin(\theta)$
(C) $x = 4\sin(\theta)$
(D) $x = 2\tan(\theta)$

5. Let the region $\mathcal{R}$ be bounded by $x = 3y + 5$ and $x = y^2 + 1$. SET UP ONLY the volume of the solid of revolution formed by revolving $\mathcal{R}$ about $y = 6$ using the **shell method**.

(A) $2\pi \int_{1}^{17} y[(3y+5) - (y^2+1)]\,dy$

(B) $2\pi \int_{-1}^{4} y[(3y+5) - (y^2+1)]\,dy$

(C) $2\pi \int_{-1}^{4} (6-y)[(3y+5) - (y^2+1)]\,dy$

(D) $2\pi \int_{-1}^{4} (y-6)[(3y+5) - (y^2+1)]\,dy$

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6. An oil tank has the shape of a right circular cone with a vertical axis. Its vertex is at the bottom, its radius is 4 feet and its height is 10 feet. It is full of oil, which weighs 50 lb/cubic feet. Using an integral, SET UP ONLY the work required to empty the tank by pumping the oil to an outlet 6 feet above the top of the tank.

(A) $50\pi \int_{0}^{10} (16-y)\left(\frac{2}{5}y\right)^2\,dy$

(B) $50\pi \int_{0}^{16} (10-y)\left(\frac{4}{5}y\right)\,dy$

(C) $50\pi \int_{0}^{10} (16-y)\left(\frac{4}{5}y\right)\,dy$

(D) $50\pi \int_{0}^{16} (10-y)\left(\frac{2}{5}y\right)^2\,dy$

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7. Which of the following statements is true?

$$A: \text{If } \lim_{n\to\infty} a_n = 1, \text{ then } \sum_{n=1}^{\infty} a_n \text{ diverges} \qquad B: \text{If } \sum_{n=1}^{\infty} b_n = 0, \text{ then } \lim_{n\to\infty} b_n = 0$$

(A) only A

(B) only B

(C) both A and B

(D) neither A nor B

8. Evaluate: $\displaystyle\int_0^{\pi} x\sin(2x)\,dx$

(A) $-\dfrac{\pi}{2}$ (B) 0 (C) $\dfrac{\pi}{2}$ (D) π

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9. Which of the following is the open interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{5n(4x-2)^n}{(-5)^n(n+2)}$$

(A) $\left(-\dfrac{3}{4}, \dfrac{7}{4}\right)$

(B) $\left(\dfrac{3}{4}, \dfrac{13}{4}\right)$

(C) $\left(-\dfrac{3}{4}, \dfrac{13}{4}\right)$

(D) None of the above

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10. Suppose you approximate $f(x) = \sin(x^2)$ by the the Maclaurin polynomial $T_2(x) = x^2$ at $x = 0.5$. The alternating series estimation theorem applied to the Maclaurin series for $f(x)$ gives an error bound equal to which of the following (i.e., fill in the blank)?

$$|f(0.5) - T_2(0.5)| \leq \quad \_\_\_\_\_$$

(A) $\dfrac{1}{32}$ (B) $\dfrac{1}{64}$ (C) $\dfrac{1}{96}$ (D) $\dfrac{1}{384}$

11. A particle traveling according to the parametric equations $x = \sin t$ and $y = \cos t$ where t represents time ($t \geq 0$)

(A) begins at the point $(0, 1)$ and travels counterclockwise around the circle $x^2 + y^2 = 1$

(B) begins at the point $(1, 1)$ and travels counterclockwise around the circle $x^2 + y^2 = 1$

(C) begins at the point $(1, 0)$ and travels clockwise around the circle $x^2 + y^2 = 1$

(D) begins at the point $(0, 1)$ and travels clockwise around the circle $x^2 + y^2 = 1$

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12. Suppose

$$a_n = \frac{n!}{(2n+1)!}, \quad b_n = \frac{3^n + 4^n}{5^n + 6^n}, \quad c_n = \frac{2^n}{3^n}$$

Which of the following statements is true?

$$A : \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{1}{2} \qquad B : \lim_{n\to\infty} \frac{b_n}{c_n} = 1$$

(A) only A

(B) only B

(C) both A and B

(D) neither A nor B

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13. The length of the curve represented by $x = (e^t) \sin t$ and $y = (e^t) \cos t$ from $t = 0$ to $t = \pi$ is

(A) $e^\pi - \sqrt{2}$

(B) $\sqrt{2}e^\pi - 2$

(C) $\dfrac{e^\pi}{\sqrt{2}}$

(D) $\sqrt{2}(e^\pi - 1)$