Math 2214, Fall 2013, Form A

1. The general solution of the system $y' = Ay$, where

$$A = \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix},$$

is

(a) $c_1 \cos(2t) \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 \sin(2t) \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

(b) $c_1 \begin{pmatrix} \sin(2t) \\ \cos(2t) \end{pmatrix} + c_2 \begin{pmatrix} \cos(2t) \\ -\sin(2t) \end{pmatrix}$.

(c) $c_1 e^{2t} \begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2 e^{-2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

(d) $c_1 e^{2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{-2t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

2. A water tank initially contains 40 gallons of water in which 200 grams of salt are dissolved. Fresh water enters the tank at a rate of 2 gallons per minute, and the well-mixed solution leaves the tank at a rate of 1.5 gallon per minute. The amount of salt in the tank, measured in grams, with time measured in minutes, is found by solving the equation

(a) $Q' = 2 - 1.5Q/40$, $Q(0) = 200$.

(b) $Q' = -1.5Q/(40 + t/2)$, $Q(0) = 200$.

(c) $Q' = 1/2 - 1.5Q/40$, $Q(0) = 200$.

(d) $Q' = -1.5 * 200/40$, $Q(0) = 200$.

3. You solve the initial value problem $y_1' = 3 + y_2^2$, $y_2' = 4t - y_1$, $y_1(1) = 1$, $y_2(1) = 3$ using the Euler method with $h = 0.05$. Then the approximation you find for $y(1.05)$ is

(a) $y_1 = 1.6$, $y_2 = 3.15$.

(b) $y_1 = 1.6$, $y_2 = 3.16$.

(c) $y_1 = 1.6$, $y_2 = 2.95$.

(d) $y_1 = 2.2$, $y_2 = 3.3$.

4. Which of the following is not a linear equation?

   (a) $y' - y = y'''$

   (b) $y''/y = \sin t$.

   (c) $y'' + |y| = 0$.

   (d) $y''' = y/t$.

5. A mass of 5 kg stretches a spring by 9.8 cm in equilibrium. Then the damping constant for critical damping is

   (a) 100 kg/sec.

   (b) $\sqrt{20}$ kg/sec.

   (c) 50 kg/sec.

   (d) $\sqrt{5}$ kg/sec.

6. The general solution of the system $y' = Ay$, where

$$A = \begin{pmatrix} 1 & -1/2 \\ 2 & -1 \end{pmatrix}$$

   is given by

   (a) $c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2 e^t \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

   (b) $c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2 (t \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix})$.

   (c) $c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2 t \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

   (d) $c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

7. If $x' = -x^3 \cos t$, and $x(0) = 1$, then $x(1)$ is

   (a) $\cos 1/(\sin 1)^3$.

   (b) $\exp(-\frac{1}{3} \sin 1)$.

   (c) $1/\sqrt{2 \sin 1 + 1}$.

   (d) $1/\sqrt{\sin 2 + 1}$.

8. A nonlinear system is given by

$$x_1' = x_1^2 - x_2^3 x_1.$$

$$x_2' = x_2 - x_1.$$

The linearization at the equilibrium point $(1, 1)$ is the system

(a)

$$y_1' = -y_2,$$
$$y_1' = -y_1 + y_2.$$

(b)

$$y_1' = 2y_1 - 3y_2,$$
$$y_1' = -y_1 + y_2.$$

(c)

$$y_1' = y_1 - 3y_2,$$
$$y_1' = -y_1 + y_2.$$

(d)

$$y_1' = 0,$$
$$y_1' = -y_1 + y_2.$$

9. Which of the following is a particular solution of the equation $y''' - y = e^t$?

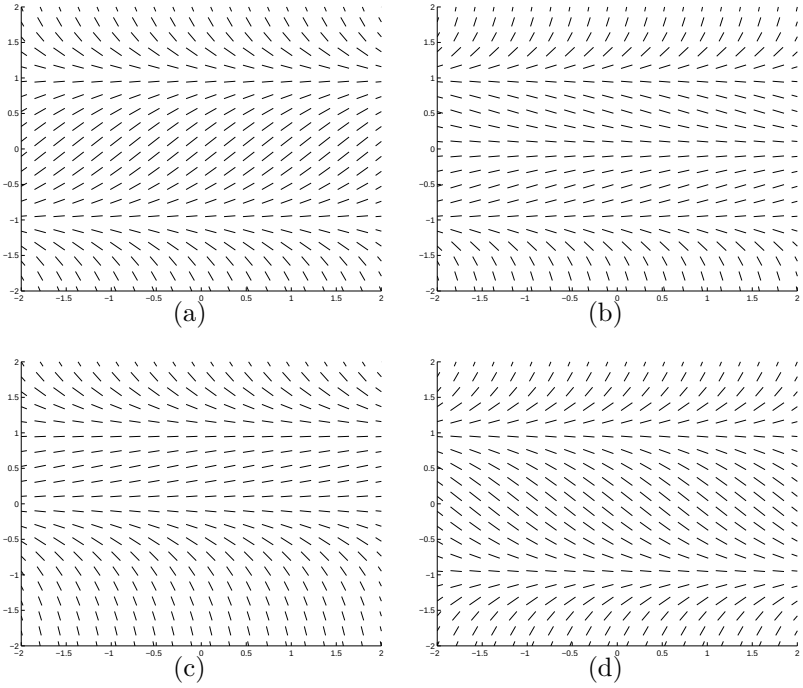
(a) $t^3 e^t/6$.

(b) $e^t/2$.

(c) $te^t/3$.

(d) $t^2 e^t/2$.

10. Which of the following is a direction field for the equation $y' = y^2 - 1$?



(a) (b) (c) (d)

11. A particular solution of the equation $y'' - y = 1/t^2$ should have the form

    (a) $u(t)e^t + v(t)e^{-t}$.

    (b) $y = A/t^2 + B/t + C$.

    (c) $y = A/t^2 + B/t^3 + D/t^4$.

    (d) $y = A/t^2 + Be^t + Ce^{-t}$.

12. For the system

$$x' = 6x + 5y,$$
$$y' = 7x + 9y,$$

    the origin is a(n)

    (a) stable focus.

    (b) unstable focus.

    (c) unstable node.

    (d) saddle.